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ABSTRACT

This elementary school mathematics program presents an activity approach with maximum involvement of students. Teaching strategies, interest center activities, appropriate games, and a measuring device are suggested for each elementary mathematics topic. Each activity is identified as part of the concrete, semiconcrete, semiabstract, or abstract stage of development for a topic. The concrete stage stresses the use of manipulative materials. The semiconcrete stage features the use of the flannel board and the magnetic board. The semiabstract stage involves the use of the chalkboard and overhead projector. The abstract phase stresses the use of paper and pencil to record responses. (MM)

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# "M⁴"

# MATH

# METHODS

# MATERIALS

# 'MAGINATION

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A PROGRAM OF MATHEMATICS FOR THE ELEMENTARY GRADES

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Wichita, Kansas

# TABLE OF CONTENTS

Letter to Teachers . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . i
Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . iii
Elementary Mathematics Skills Sheet. . . . . . . . . . . . . . . . . . . . . . . . . iv
Elementary Mathematics Program . . . . . . . . . . . . . . . . . . . . . . . . . . . viii
Flow Chart of Concept Development. . . . . . . . . . . . . . . . . . . . . . . . . . x
Schematic Diagram of Elementary Mathematics Program. . . . . . . . . . . . . . . . . xi
Comparisons. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
Positional Relationships . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15
Patterning . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 22
Counting . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 34
Set Recognition. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 43
One to One Matching. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 50
Non-Equivalent Sets. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 55
Recognition of Numerals. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 57
Number Words . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 62
Forming Sets of Numerals . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 67
Matching Numeral With Set. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 74
Writing of Numerals. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 79
Order of Numerals. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 83
Number Comparisons . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
Conservation of Sets . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 108
Joining Sets . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 113
Addition Facts . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 118
Separating Sets. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 131
Subtraction Facts. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 135
Place Value. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 141
The Numbers 100-999. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 154
Regrouping . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 159
Addition of Whole Numbers. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 171
Subtraction of Whole Numbers . . . . . . . . . . . . . . . . . . . . . . . . . . . . 180
Equivalent Sets and Subsets. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 189
Multiplication Facts . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 196
Symbols Classification of Numbers. . . . . . . . . . . . . . . . . . . . . . . . . . 214
Roman Numerals . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 217
Partitioning Sets. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 222
Division Facts . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 228
Multiplication of Whole Numbers Beyond Basic Facts . . . . . . . . . . . . . . . . . 234
Division of Whole Numbers. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 237
Fractions. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 245
Addition of Fractions. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 259
Subtraction of Fractions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 265
Multiplication and Division of Fractions . . . . . . . . . . . . . . . . . . . . . . 268
Decimal Fractions and Decimals . . . . . . . . . . . . . . . . . . . . . . . . . . . 283
Measurement. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 291
Measurement of Capacity. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 292
Measurement of Length. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 297
Measure of Time. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 302
Metric Measurement . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 308
Measurement of Money . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 309
Measurement of Geometric Figures . . . . . . . . . . . . . . . . . . . . . . . . . . 315
Interpretation of Graphs . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 320
Construction of Graphs . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 323
Mathematical Applications. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 330
Integers . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 331

Dear Teacher:

This book is not a guide, it is a program of mathematics for the elementary grades. It is an activity approach with maximum involvement of the students. Elementary school students frequently become discouraged in learning mathematics because of a general inability to relate mathematical concepts to the facts of their daily lives and, at times, because they are asked to move too quickly from one concept to another. They soon become convinced that they cannot learn, and the teacher, in his frustration, may reinforce that perception in his students. Educational innovations and modifications of existing programs should, therefore, be made early enough to prevent this discouragement on the part of the student, and to provide him with the necessary challenge to keep his motivation high.

Students need to use more concrete, manipulative activities than are normally provided in a school program. The ability to physically represent and manipulate numbers gives the student confidence and enables him to make a particular mathematical concept his own.

The elementary mathematics program involves a four phase approach in the teaching of mathematics. Phase I, the concrete stage, stresses the use of manipulative materials. Phase II, the semiconcrete stage, features the use of the flannel board and the magnetic board. Phase III, the semi-abstract stage, involves the use of the chalkboard and overhead projector. Phase IV, the abstract stage, emphasizes the use of paper and pencil to record responses.

This book contains teaching strategies for the first three phases, and for each mathematical concept. The program also provides interest center activities, appropriate games, and a measuring device for each concept. Workbook pages and worksheets should be utilized for the abstract stage in teaching concepts.

Teachers should remember that the skill sheet reflects what a child should learn over a three year period of time (K-6). It is important that the teacher spot check the mastery level of those skills that have been recorded or mastered by previous teachers. Do not try to reteach all the concepts previous to your grade level, merely those that students have not mastered. This review should be accomplished as quickly as possible.

Each child should begin his study of concepts at a level equal to his ability.

Although this book is designed to be a program of mathematics for the elementary grades, the teacher should be encouraged to improvise with any materials that are available and to devise additional activities tailored to specific classroom needs.

Ed Schroeder
Mathematics Coordinator
Wichita Public Schools

# INTRODUCTION

The trend in mathematics is away from the text and cookbook recipe kind of mathematics and toward extensive laboratory work and open-ended experiments. Students are encouraged to move forward as rapidly as possible on an individual basis, with the more able students being encouraged to explore related projects. The imaginative and innovative teacher is freed from the tight textbook approach to mathematics.

Children must, from the beginning, be exposed to the structure of mathematics. They find concepts intensely interesting, can discover and make use of patterns and relationships, can think creatively and analytically, and are stimulated by and interested in new mathematical topics. Also, the learning process is shorter and more effective when it is based upon a conceptual approach that emphasizes the discovery of ideas.

When the actual experiences of children are used as the source of classroom activities, teachers will have little difficulty in making the work interesting and closely related to the needs of individual learners.

Students cannot learn by being told. They must see, hear, feel, smell, and taste for themselves. The terms hot, sharp, and wet have no meaning for children until they actually experience the physical sensations associated with each word.

Piaget[1] emphasizes two things about active learning. First, a child must be allowed to do things over and over again and thus reassure himself that what he has learned is true. Second, this practice should be enjoyable. Anyone who has observed the look of sheer joy that enlivens the face of a young child when he succeeds in opening a door, standing up on skates, or solving a puzzle, will support Piaget on this point. Unfortunately, too many adults do not regard this as learning. Many still equate learning with work, and work with discomfort or unpleasantness. In fact, one of the most difficult problems for progressive teachers to overcome is the suspicion that many parents have for programs which their children obviously enjoy. "If they like school that much, they can't be working hard enough to learn anything."

This program is not one where children memorize a vast number of facts. It is a program designed to teach children exactly what certain facts mean. Many children have no understanding of what is going on in mathematics. They may be able to memorize statements such as $7 + 3 = 10$, or $10 - 3 = 7$ without the slightest idea of what those statements really mean. For these children, mathematics is an unending mystery. It will remain a mystery unless they are taught in a logical and precise manner exactly what mathematical statements mean.

---

[1] Edith E. Biggs and James R. Maclean. Freedom to Learn. Redding, Massachusetts: Addison Wesley, 1969.

SCHOOL________________________________ NAME________________________________

Shade in the circle to indicate the student has demonstrated concept mastery.

GRADE K 1 2 3 4 5 6 (Circle current grade level)

1. COMPARISONS
   - ○ Large and small
   - ○ Tall and short
   - ○ Heavy and light
   - ○ Long
   - ○ More and less

2. POSITIONAL RELATIONSHIPS
   - ○ Under and over
   - ○ Top and bottom
   - ○ Front and back
   - ○ High and low
   - ○ Far and near
   - ○ Between
   - ○ Inside and outside

3. PATTERNING
   - ○ Two shapes
   - ○ Two colors
   - ○ Three shapes
   - ○ Three colors

4. COUNTING BY ONES
   - ○ Rational to 10
   - ○ Rational to 20
   - ○ Rational to 100
   - ○ Rote to 20
   - ○ Rote to 100
   - ○ Serial to 10
   - ○ Ordinals to fifth
   - ○ Ordinals to tenth
   - ○ Ordinals to twentieth

5. COUNTING BY TWOS
   - ○ Rational to 20
   - ○ Rote to 50

6. COUNTING BY FIVES
   - ○ Rational to 50
   - ○ Rote to 100

7. COUNTING BY TENS
   - ○ Rote to 100

8. COUNTING BY HUNDREDS
   - ○ Rote to 1000

9. RECOGNITION OF SETS
   - ○ (0-5)
   - ○ (0-10)

10. ONE-TO-ONE MATCHING
    - ○ Equivalent
    - ○ Non-equivalent
    - ○ One to ten
    - ○ One to fifty
    - ○ One to one hundred
    - ○ One to one thousand

11. RECOGNITION OF NUMERALS
    - ○ (0-10)
    - ○ (0-50)
    - ○ (0-100)
    - ○ (0-500)
    - ○ (0-1000)

12. NUMBER WORD RECOGNITION
    - ○ One to ten
    - ○ One to fifty
    - ○ One to one hundred
    - ○ One to five hundred
    - ○ One to one thousand

13. FORMING SETS FOR NUMERALS
    - ○ (0-10)
    - ○ (0-20)

14. MATCHING NUMERALS WITH SETS
    - ○ (0-10)
    - ○ (0-20)

15. WRITING NUMERALS
    - ○ (0-10)
    - ○ (0-50)
    - ○ (0-100)
    - ○ (0-500)
    - ○ (0-1000)

16. ORDER OF NUMERALS
    - ○ (0-10)
    - ○ (0-50)
    - ○ (0-100)
    - ○ (0-500)
    - ○ (0-1000)
    - ○ Number line

### 17. NUMBER COMPARISONS

- ○ Before
- ○ After
- ○ Between
- ○ One more
- ○ One less
- ○ Less than
- ○ More than
- ○ Even number
- ○ Odd number
- ○ Prime number
- ○ Number patterns

### 18. CONSERVATION OF SETS

- ○ (1-6)
- ○ (1-12)
- ○ (1-18)

### 19. JOINING SETS

- ○ (1-6)
- ○ (1-12)
- ○ (1-18)

### 20. ADDITION FACTS

- ○ (1-6)
- ○ (1-12)
- ○ (1-18)
- ○ Number line addition

### 21. SEPARATING SETS

- ○ (1-6)
- ○ (1-12)
- ○ (1-18)

### 22. SUBTRACTION FACTS

- ○ (1-6)
- ○ (1-12)
- ○ (1-18)
- ○ Number line subtraction

### 23. PLACE VALUE

- ○ Ones
- ○ Tens
- ○ Hundreds
- ○ Thousands
- ○ Millions
- ○ Billions
- ○ Expanded notations

### 24. RENAMING

- ○ Ones ↔ Tens
- ○ Tens ↔ Hundreds
- ○ Hundreds ↔ Thousands

### 25. ADDITION OF WHOLE NUMBERS

- ○ Missing addend
- ○ 3 addends
- ○ 2 digit addends (no renaming)
- ○ 2 digit addends (renaming)
- ○ 3 digit addends (no renaming)
- ○ 3 digit addends (renaming)
- ○ 4 digit addends
- ○ Worded problems

### 26. SUBTRACTION OF WHOLE NUMBERS

- ○ 2 digit (no renaming)
- ○ 2 digit (renaming)
- ○ 3 digit (no renaming)
- ○ 3 digit (renaming)
- ○ 4 digit
- ○ Worded problems

### 27. EQUIVALENT SUBSETS

- ○ Twos
- ○ Threes
- ○ Fours
- ○ Fives
- ○ Sixes
- ○ Sevens
- ○ Eights
- ○ Nines

### 28. MULTIPLICATION FACTS

- ○ To 3x3
- ○ To 4x4
- ○ To 5x5
- ○ To 7x7
- ○ To 9x9
- ○ Number line multiplication

### 29. SYMBOLS

- ○ (+) ○ (-)
- ○ (x) ○ (÷)
- ○ (<) ○ (>)
- ○ ($) ○ (¢)
- ○ (%) ○ (π)
- ○ Triangle
- ○ Square
- ○ Rectangle
- ○ Circle
- ○ (o) ○ (=)
- ○ Point ○ Segment
- ○ Ray ○ Line
- ○ ⊥ lines
- ○ Parallel lines
- ○ Intersecting lines
- ○ Angle ○ Right angle
- ○ Right triangle
- ○ Quadrilateral
- ○ Parallelagram
- ○ Rhombus

### 29. SYMBOLS, cont.

- ○ Pentagon
- ○ Hexagon
- ○ Octagon
- ○ Cube
- ○ Cone
- ○ Cylinder
- ○ Sphere
- ○ Prism

### 30. CLASSIFICATION OF NUMBERS

- ○ Composite
- ○ Prime
- ○ Common fractions
- ○ Decimals

### 31. ROMAN NUMERALS

- ○ (I-X) ○ (X-L)
- ○ (L-C) ○ (C-M)

### 32. PARTITIONING SETS

- ○ By twos
- ○ By threes
- ○ By fours
- ○ By fives
- ○ By sixes, sevens
- ○ By eights, nines

### 33. DIVISION FACTS

- ○ To 9÷3
- ○ To 16÷4
- ○ To 25÷5
- ○ To 49÷7
- ○ To 81÷9
- ○ Number line division

### 34. MULTIPLICATION OF WHOLE NUMBERS

- ○ 10 as a multiplier
- ○ 2 digit by 1 digit
- ○ 2 digit by 2 digit
- ○ 3 digit by 2 digit
- ○ 100 as a multiplier
- ○ 3 digit by 3 digit
- ○ Worded problems

### 35. DIVISION OF WHOLE NUMBERS

- ○ 1 digit divisor (no remainder)
- ○ 1 digit divisor (remainder)
- ○ 10 as a divisor
- ○ 2 digit divisor
- ○ 100 as a divisor
- ○ 3 digit divisor
- ○ Worded problems

### 36. FRACTIONS

- ○ (1/2) ○ (1/4)
- ○ (3/4) ○ (1/3)
- ○ (2/3)
- ○ Regions
- ○ Sets
- ○ Fractional parts of one
- ○ Renaming in simplist form
- ○ Renaming in equivalent form
- ○ Mixed numeral ↔ fractions
- ○ Order on number line

### 37. ADDITION OF FRACTIONS

- ○ Like denominators with sums less than one
- ○ Like denominators with sums greater than one
- ○ Unlike denominators
- ○ Mixed numerals

### 38. SUBTRACTION OF FRACTIONS

- ○ Common fractions with like denominators
- ○ Common fractions with unlike denominators
- ○ Whole number Common fractions
- ○ Mixed numeral Common fractions
- ○ Mixed numeral mixed numeral

### 39. MULTIPLICATION OF FRACTIONS

- ○ Common with 1 as a numerator
- ○ Common fractions
- ○ Whole number x common
- ○ Improper x common
- ○ Improper x Improper
- ○ Mixed number x common
- ○ Mixed number x mixed number

### 40. DIVISION OF FRACTIONS

- ○ Whole number ÷ common
- ○ Common fractions
- ○ Improper ÷ common

### 40. DIVISION OF FRACTIONS, cont.

○ Improper ÷ improper
○ Mixed number ÷ common
○ Common ÷ mixed number
○ Whole number ÷ mixed number

### 41. DECIMAL FRACTIONS

○ Tenths
○ Hundredths
○ Thousandths
○ Order on number line

### 42. DECIMALS

○ Place value
○ Expanded notation
○ Addition
○ Subtraction
○ Multiplication
○ Division

### 43. ENGLISH MEASUREMENT

○ Cup ○ Pint
○ Quart ○ Gallon
○ Inch ○ Foot
○ Yard
○ Ounce ○ Pound

### 44. MEASUREMENT OF TIME

○ Hour
○ Half-hour
○ Quarter time
○ Exact time

### 45. METRIC MEASUREMENT

○ Metre ○ Litre
○ Gram
○ Centi- cm, cg, cl
○ Deci- dm, dg, dl
○ Deka- dam, dag, dal
○ Kilo- km, kg, kl
○ Milli- mm, mg, ml
○ Hecto- hm, hg, hl

### 46. MEASUREMENT OF MONEY

○ Penny ○ Nickel
○ Dime ○ Quarter
○ Half dollar
○ Dollar
○ Knows values
○ Makes change

### 47. MEASUREMENT OF GEOMETRIC FIGURES

○ Angles in degrees
○ Perimeter of polygons
○ Circumference of circles
○ Area of rectangles
○ Area of triangles
○ Area of circles
○ Volume of prism

### 48. INTERPRETATION OF GRAPHS

○ Charts and tables
○ Pictorial
○ Line
○ Bar
○ Circle
○ Number pairs

### 49. CONSTRUCTION OF GRAPHS

○ Charts and tables
○ Pictorial
○ Line
○ Bar
○ Circle
○ Number pairs

### 50. MATHEMATICAL APPLICATIONS

○ Compute averages
○ Compute percentages
○ Use formulas
○ Ratio--Proportion problems
○ Solve equations
○ Rounding numbers
○ Congruent figures
○ Story problems
○ Money story problems

### 51. INTEGERS

○ Positive
○ Negative
○ Opposites
○ Addition
○ Subtraction

## ELEMENTARY MATHEMATICS PROGRAM

Teachers are responsible for:

(1) Teaching math to all the children in their classroom and seeing that each child develops his math potential to its maximum.

(2) Teaching → evaluating → reteaching and re-evaluation for concept mastery.

(3) Grouping their children for math instruction. (Each group could be working on a different concept or the same concept at different levels.)

(4) Informing the lab aide in writing (on Friday) of the concept(s) to be worked on with each lab group for the coming week, and suggesting possible lab activities. (Remember the aide is not a certificated teacher.)

(5) Participating in the inservice training activities provided by the program.

(6) Keeping their children's skill sheets current, using only the designated markings.

(7) Using the adopted math text only as a supplement to the Elementary Mathematics Program.

(8) Identifying and providing additional instructional time for those children in the Title I schools who have been identified as having mathematical concept deficiencies.

# ELEMENTARY MATHEMATICS PROGRAM

Job Description:

The Elementary Math Project Aide is responsible for assisting project teachers in improving the math skills of their pupils. The Elementary Mathematics Project Aide is responsible to the building principals. The aide must adhere to building policies as set down by the building principal. The aides' principal responsibilities are:

(1) Administer the oral pretest and post test to each kindergarten and first grade child.

(2) Construct visual aids for use in the classroom when requested by teacher or consultant. These visual aids would include: interest center devices, games for motivational drill, overhead projector transparencies, and various other teaching aids.

(3) Reproduce test materials as requested by teachers for classroom use.

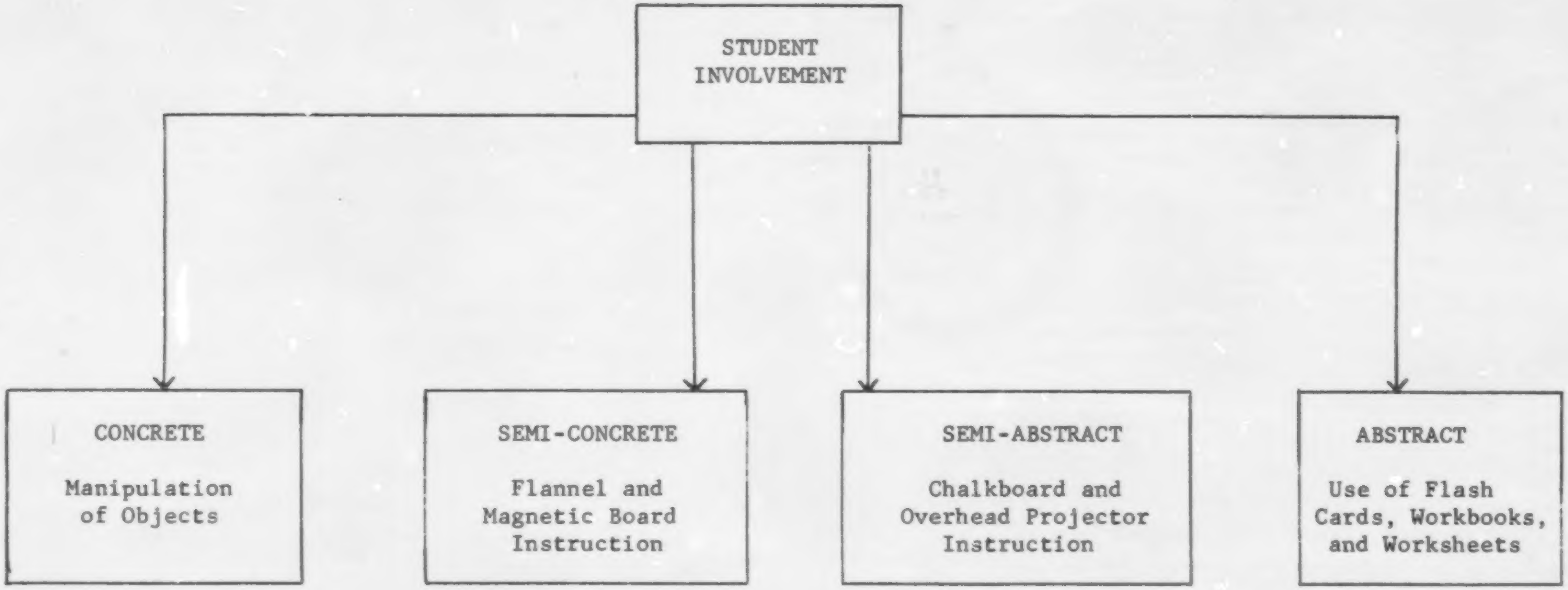
(4) Give oral tests to children for concept mastery when requested by teachers.

(5) Work with students in the math lab as directed by the teacher. (Approximately 75% of the aides' time will be utilized working with children in the lab.)

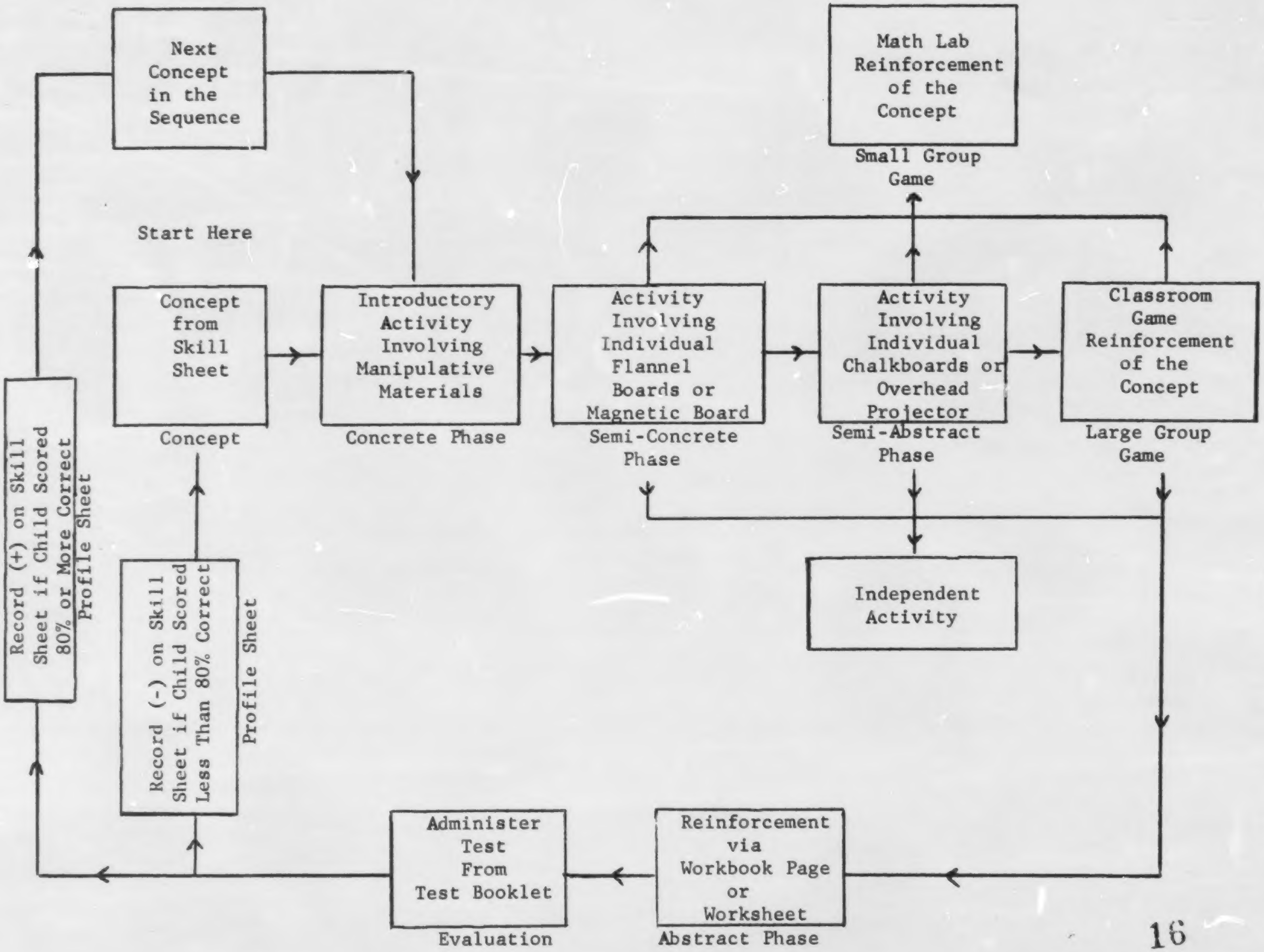
(6) Construct the math games that are to be used in the lab.

(7) Conduct tours of the math lab and demonstrate the various games to nonproject teachers who are visiting the math lab.

## FLOW CHART OF CONCEPT DEVELOPMENT


STUDENT INVOLVEMENT
CONCRETE
Manipulation of Objects
SEMI-CONCRETE
Flannel and Magnetic Board Instruction
SEMI-ABSTRACT
Chalkboard and Overhead Projector Instruction
ABSTRACT
Use of Flash Cards, Workbooks, and Worksheets

SCHEMATIC DIAGRAM OF ELEMENTARY MATHEMATICS PROGRAM


Next Concept in the Sequence
Math Lab Reinforcement of the Concept
Small Group Game
Start Here
Concept from Skill Sheet
Concept
Introductory Activity Involving Manipulative Materials
Concrete Phase
Activity Involving Individual Flannel Boards or Magnetic Board
Semi-Concrete Phase
Activity Involving Individual Chalkboards or Overhead Projector
Semi-Abstract Phase
Classroom Game Reinforcement of the Concept
Large Group Game
Independent Activity
Record (+) on Skill Sheet if Child Scored 80% or More Correct Profile Sheet
Record (-) on Skill Sheet if Child Scored Less Than 80% Correct Profile Sheet
Administer Test From Test Booklet
Evaluation
Reinforcement via Workbook Page or Worksheet
Abstract Phase

# COMPARISONS

## a. Comparison of objects - by inspection

### Introduction

Children should be given frequent opportunities for comparing objects with respect to such properties as: size, height, weight, and age. They should be encouraged to see that, at this stage, they are comparing the objects in sets, according to some common property (color, weight, age, etc.). At a later time they will be expected to compare sets of objects according to the one property common to all sets (the number of objects they contain). At that time, the color, size, weight, height, and shape of the objects in the sets will be irrelevant when considering the basic number property. They will then learn that the number of objects contained in a given set is called the "cardinal number" of the set.

The terms "equal to" and "same as" may now be introduced. When comparing two sets which do not have the same number of elements, the instructor is laying the ground work for the concept of order of numbers. Terms such as "more than", "less than", "most", "fewest", etc., may be introduced at this time. Do not introduce the symbols >, =, < at this stage.

## CLASSROOM ACTIVITIES

Concept: Comparisons - size

Stage of Development: Concrete

Equipment: Three chairs of various sizes

Activity: The children are seated in a circle (on the floor). The three chairs are placed in the center of the circle. A student comes forward and sits in the largest chair. Then another student sits in the smallest one.

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Stage of Development: Concrete

Equipment: Three balls of different sizes

Activity: The balls should be placed so that all can see them. A student chooses the largest ball and hands it to the teacher. Another student hands the teacher the smallest one.

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Stage of Development: Concrete

Equipment: Three blocks of different sizes

Activity: Blocks are placed so all can see them. One student chooses the largest block, then someone else chooses the smallest one.

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Concept: Comparisons - size

Stage of Development: Semiconcrete

Equipment: Flannel cutouts showing three sizes of objects, flannel board

Activity: The teacher places the cutouts on the flannel board so all the children may see and compare them. A student is chosen to show the class the largest cutout. Another child may show the smallest cutout. This should be repeated, using several different objects in several different positions.

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Stage of Development: Semiconcrete

Equipment: Individual flannel boards, different sizes of the same object

Activity: The children are told to arrange the objects on their flannel boards, going from largest to smallest. This activity should be closely supervised by the teacher. The students then remove all but the largest object from their boards. The smallest object can then be placed by the largest.

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Concept: Comparisons - size

Stage of Development: Semiabstract

Equipment: Overhead projector, flannel objects of different sizes

Activity: The teacher places the objects on the overhead projector, so that images of them are shown on the screen. A child comes forward and points to the largest object. A different child points out the smallest object. This should be repeated with several different objects.

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Stage of Development: Semiabstract

Equipment: Overhead projector, penny, dime, and quarter

Activity: All coins are placed on the overhead projector so that different-sized images are shown on the screen. One student points out the largest object, another student points out the smallest one, and still another arranges them from largest to smallest.

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Stage of Development: Semiabstract

Equipment: Pictures of animals and objects, showing different sizes

Activity: The teacher shows pictures of a whale and a butterfly. The children are asked to tell which is larger and which is smaller. Then, pictures of a lion, a cat, and a mouse are shown. The class determines which is largest and which is smallest. Pictures of a giraffe, a calf, and a boy are shown. The students determine which is largest and which is tallest.

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Concept: Comparisons - height

Stage of Development: Concrete

Equipment: Children

Activity: The teacher chooses three children of various heights to stand next to each other in front of the class. A student is asked to name the tallest child and then the shortest one. Another pupil arranges the children from the shortest to the tallest. A child may also compare his height with that of other children, i.e. - "I am taller than ________. I am shorter than__________."

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Stage of Development: Concrete

Equipment: Three objects of different heights (e.g., dolls, sticks, cans, etc.)

Activity: Three objects are placed next to each other so the children have a basis for comparison. The teacher asks a child to point out the tallest object. Another child finds the shortest object. The objects may also be arranged from tallest to shortest.

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Concept: Comparisons - height

Stage of Development: Semiconcrete

Equipment: Flannel board, three flannel objects of different heights (e.g., people, trees, poles, etc.)

Activity: Objects that are similar, but have different heights, are placed on the flannel board in such a way that the class has a basis for comparison. One member of the class chooses the tallest and shortest objects and then arranges all of them from tallest to shortest. This can be repeated with different flannel objects.

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Stage of Development: Semiconcrete

Equipment: Magnetic board, three magnetized shapes which are similar but have different heights

Activity: The objects are placed on the magnetic board. One student picks out the tallest object. Someone else picks out the shortest one. Another child can arrange them from the shortest to the tallest.

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Concept: Comparisons - height

Stage of Development: Semiabstract

Equipment: Three sticks of different lengths or three flannel objects of different heights, overhead projector

Activity: The objects that are to be compared should be placed on the overhead projector in such a way that the children can make a comparison by sight. One child is chosen to tell the class which object is the tallest and which is the shortest.

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Stage of Development: Semiabstract

Equipment: Overhead projector, transparencies showing objects of different heights, overhead water-base pen

Activity: A transparency is placed on the overhead projector. Then, a child marks an "x" on the tallest object on the transparency. Another child marks an "o" on the shortest object.

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Concept: Comparisons - weight

Stage of Development: Concrete

Equipment: Two objects of the same size, but with obviously different weights when compared (e.g., wooden ball and rubber ball, empty one-pound coffee can and one full of sand, etc.)

Activity: The two objects are placed so all in the class can see them. A child lifts each one and then points out the one that is heavier. A different student repeats this process to determine the lighter object. Several different objects (using a different pair) should be used to give the students a variety of experiences.

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Stage of Development: Concrete

Equipment: Two objects of different sizes, but with the smaller one being the heavier (e.g., piece of lead compared to styrofoam ball, etc.)

Activity: The two objects are placed so all can see them. One child examines each object by lifting it. The student then points out the heavier object. Another student repeats the examination; this time the lighter object is selected.

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Stage of Development: Concrete

Equipment: Bathroom scales, children, counting blocks

Activity: Each child is weighed and his weight is recorded. A child's weight can be represented by forming a set of blocks, using one block for each five pounds of his weight. (The teacher should not worry about any pounds over the five-pound units.) The blocks are stacked on top of each other to form a column. The columns must be close together so the children will have a basis for comparison. The child that weighs the most will be the one with the tallest column of blocks. The one that weighs the least will be the one with the shortest column of blocks. Children that have columns of equal height, weigh the same. If the blocks are arranged in rows, the longest row will represent the heaviest child; the shortest row will represent the lightest child; and rows of the same length will represent children of equal weight.

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Concept: Comparisons - weight

Stage of Development: Semiconcrete

Equipment: Flannel board, congruent flannel squares, bathroom scales, children

Activity: Flannel squares are used to represent the weight of the child, and the activity then proceeds as described in the section immediately preceding this one.

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Stage of Development: Concrete

Equipment: A balance if constructed, as shown in the game book.

Activity: The balance scales can be used to compare the weight of two objects. The objects are shown to the class. The children guess as to which is the heavier, or lighter. The balance scale will determine the answer to the comparison. This activity can be repeated with four objects, two of which have the same weight. The children should always make a guess before the experiment is performed.

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Concept: Comparisons - length

Stage of Development: Concrete

Equipment: Three objects with obviously different lengths (e.g., boards, sticks, dowel rods, pencils, etc.)

Activity: The objects being compared are placed so all can see them. The children then choose the longest, the shortest, and arrange them from shortest to longest. They should also be able to arrange them from longest to shortest.

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Concept: Comparisons - length

Stage of Development: Semiconcrete

Equipment: Flannel boards, flannel shapes of different lengths (e.g., rectangles, squares, rods, etc.)

Activity: The shapes are placed on the flannel board for all to see. The children are given a chance to compare the lengths. One child selects the longest one and the shortest one. The students will then arrange the flannel shapes from shortest to longest, and from longest to shortest.

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Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic objects of different lengths

Activity: The teacher will follow the instructions given in the previous activity.

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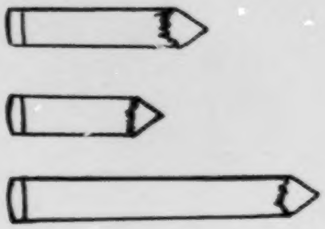
Concept: Comparisons - length

Stage of Development: Semiabstract

Equipment: Overhead projector, transparencies showing objects of different lengths, pencils of different lengths, overhead projector pen

Activity 1: The pencils are placed on the overhead projector, using the following diagram:



The class determines which pencil is the longest and which is the shortest. One child may then arrange them from longest to shortest, and from shortest to longest.

Activity 2: The teacher draws a line segment on the overhead projector. A pen is then given to one of the students, and he is asked to draw a line segment longer than the one the teacher has drawn. Someone else will draw a line segment smaller than the two already on the overhead projector. This activity can be repeated, giving several children a chance to participate.

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Stage of Development: Semiabstract

Equipment: Chalkboard, chalk

Activity: Several children should go to the chalkboard. Individual chalkboards may be used, if available. The children are instructed to draw three line segments of different lengths. They are then told to draw an "x" on the longest segment and an "o" on the smallest segment.

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Concept: Comparisons - volume

Stage of Development: Concrete

Equipment: Cups

Activity #1: Place an equal volume of snow and water into two cups. Require the children to guess which container will have the larger amount of water after the snow has melted. Allow the snow to melt and compare volumes.

Activity #2: Repeat the above experiment but use packed down snow in one cup and loose snow in the other. Allow snow to melt. Then compare the amounts of water.

Activity #3: Repeat the above activity using ice cubes and crushed ice.

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Stage of Development: Concrete

Equipment: 12 Identical containers (baby food jars, pill bottles, etc.)
4 Different commodities (rice, beans, etc.)

Activity: Take each commodity and place different amounts in three containers. Mix up the containers. Ask the children to match the containers according to commodity and then arrange them according to amounts. Questions similar to the following should be asked: Which container has the most rice? Which container has the least corn?, etc.

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Concept: Comparisons - volume

Stage of Development: Concrete

Equipment: 3 Identical clear plastic containers
Package--rice, beans, etc.

Activity: Fill the containers with the commodity so that it is obvious that they have different amounts. By observation, have the children determine the container having the most of the commodity and the least.

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Concept: Comparisons - volume

Stage of Development: Semiconcrete

Equipment: Demonstration flannel or magnetic board
Cutouts for flannel or magnetic boards of identical containers indicating different volumes

Activity #1: Place the cutouts on the board. A child is required to select the one containing the least or the most volume. Repeat with different cutouts.

Activity #2: Place one cutout on the board. A child is required to place a cutout on the board that has either more or less volume.

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Concept: Volume

Stage of Development: Semiabstract

Equipment: Overhead projector

Activity: Two containers are drawn on the overhead projector, one to show a given amount. Require a child to fill in the other container with either more or less than the given one.

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## CLASSROOM ACTIVITIES

Concept: Comparison of sets - by inspection

Stage of Development: Concrete

Equipment: Three display trays, counters (sticks, blocks, discs, etc.)

Activity: The three display trays are placed on a table in front of the class. They must contain obviously different numbers of elements (counters). The children then react to the following directions:

1. "Choose the tray with the fewest counters."
2. "Choose the tray with the most counters."
3. "Arrange the trays from left to right going from the fewest to the most."
4. "Arrange the trays from left to right going from the most to the fewest."

This can be repeated with different sizes of sets and counters.

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Concept: Comparison of sets - by inspection

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel objects

Activity: Three sets are formed on the flannel board with each set having an obviously different number of elements. The following questions should be asked:

1. "Which set contains the most objects?"
2. "Which set contains the fewest objects?"

The activity can be repeated with three new sets. The teacher must not allow the children to count the elements.

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Stage of Development: Semiconcrete

Equipment: Flannel board, flannel shapes, magnetic board, magnetized shapes

Activity: Four sets are formed on the flannel or magnetic boards. Two of the sets should have the same number of elements, while the other two will have an obviously different number of elements. The following questions are asked:

1. "Which set contains the most objects?"
2. "Which two sets contain the same number of objects?"
3. "Which set contains the fewest objects?"

The children must not be allowed to count the elements.

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**Concept:** Comparison of sets - by inspection

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector (coins, discs, shapes - for showing images)

**Activity:** Four sets are formed on the overhead projector. Two of the sets will have the same cardinal number and the other two will have an obviously different number of elements. These questions should be asked:

1. "Which set has the most objects?"
2. "Which two sets have the same number of objects?"
3. "Which set has the fewest objects?"

The children are not allowed to count the elements.

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**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, several transparencies showing sets (some equal and others with a different number of elements)

**Activity:** The teacher cuts out the sets from the transparencies. Four sets are placed on the overhead projector. The children then respond to the following directions:

1. "Show me the largest set." (One having the most elements.)
2. "Show me the smallest set."
3. "Show me the two sets that have the same number of objects."
4. "Arrange the sets from the fewest to the most."
5. "Arrange the sets from the most to the fewest."

This can be repeated with different sets.

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Concept: Comparisons - age

Stage of Development: Semiconcrete

Equipment: Flannel board, pictures of the faces of people of various ages (backed with sandpaper or velour paper so they will adhere to the flannel board)

Activity #1: A picture of an adult and a child is placed on the flannel board. The teacher asks someone to identify the older person. Two different pictures are placed on the flannel board and a child is asked to identify the younger person.

Activity #2: Place pictures of persons representing three or more different ages on the flannel board (baby, child, adult, older adult). Ask a child to identify the youngest and oldest person and arrange their pictures according to their ages.

Activity #3: Place one of the pictures on the flannel board. Ask a child to place a picture of a person that is either younger or older on the flannel board.

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Stage of Development: Semiconcrete

Equipment: Magentic board, pictures of the faces of people of various ages (backed with a piece of magnet so they will adhere to the magentic board)

Activity: Repeat all of the above activities using the magnetic board.

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Stage of Development: Semiabstract

Equipment: Overhead projector, transparencies of the faces of people of various ages

Activity: Repeat all of the flannel board activities using the overhead projector and transparencies.

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**Concept:** Comparisons - age

**Stage of Development:** Abstract

**Equipment:** None

**Activity:** One student calls on another to compare ages. This comparison is to determine who is the older. Two other students may compare ages to determine who is the younger. The children can name younger brothers and sisters; also, they may name older brothers and sisters. The teacher then asks this question, "Who is older, you or your dad?" Or perhaps this question may be asked, "Who is the younger, you or your mother?"

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**Concept:** Comparisons - age

**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, pictures of adults and children (backed with velour paper so they will adhere to the flannel board)

**Activity:** A picture of an adult and a child is placed on the flannel board. The teacher asks someone to identify the older person. Two different pictures of adults and children are then placed on the flannel board. A child is asked to choose the youngest person. Pictures of a grown man, a boy, and a small baby are displayed on the flannel board. These pictures must be arranged from youngest to oldest.
(Note: The above activity could also be used for animals of different ages, such as, horse and colt, cow and calf, etc.)

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**Concept:** Comparisons - age

**Stage of Development:** Semiabstract

**Equipment:** Magnetic board, pictures of adults and children, horses and colts, cows and calves, cats and kittens, etc., small magnets to hold pictures on the magnetic board

**Activity:** The teacher places pictures of an adult and a child on the magnetic board for all to see. One pupil selects the older person. Another one finds the younger person.
The above activity can be repeated with animals of different ages.

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## POSITIONAL RELATIONSHIPS

### Introduction

Terms of location, such as "between", "in front of", and "behind", involve a form of comparison, since they require the establishment of a relationship between two objects.

The importance of these terms will become apparent when considering the order of numerals. The numeral 5 can be described as the numeral "before" the numeral 6; the numeral 5 can be described as the numeral "after" the numeral 4; or, the numeral "between" the numerals 4 and 6.

The child needs a variety of experiences concerning positional concepts as related to objects.

NOTE: There are no activities listed for the concepts of "far" and "near" in the book. The teacher should introduce these concepts through verbal interaction with the children. (e.g., Who is nearest the door?, Which is farthest from the child, the swings on the playground or the pencil sharpener in the classroom?)

## CLASSROOM ACTIVITIES

**Concept:** Positional relationships

**Stage of Development:** Concrete

**Equipment:** Children, chairs

**Activity:** The teacher will give the following directions to all the children concerning their relative positions with their chairs:

1. "Sit on your chair."
2. "Stand in back of your chair."
3. "Stand in front of your chair."
4. "Place your foot under your chair."
5. "Stand on the left side of your chair."
6. "Stand behind your chair."
7. "Stand on the right side of your chair."
8. "Stand on your chair."

To make this more complicated, three chairs of different sizes can be placed in front of the class. The following instructions should be given:

1. "Stand in front of the largest chair."
2. "Stand behind the smallest chair."
3. "Sit on the chair that is in the middle."
4. "Sit behind the smallest chair."
5. "Sit on the largest chair."
6. "Sit on the smallest chair."

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**Stage of Development:** Concrete

**Equipment:** Chair, toy car

**Activity:** The children are asked to respond to the following directions:

1. "Place the car on the chair."
2. "Place the car below the chair."
3. "Place the car under the chair."
4. "Place the car behind the chair."
5. "Place the car beneath the chair."
6. "Place the car in back of the chair."

Place two chairs in front of the children:

7. "Place the car between the chairs."

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**Stage of Development:** Concrete

**Equipment:** Three chairs of different sizes, three shapes: square, triangle, and circle.

**Activity:** The children will respond to directions, such as the following:

1. "Put the triangle on the big chair."
2. "Put the circle behind the small chair."
3. "Put the square in front of the small chair."
4. "Put the circle behind the middle-sized chair."
5. "Put the square in front of the big chair."
6. "Put the triangle at the right side of the small chair."
7. "Put the square at the right side of the big chair."
8. "Put the triangle on the middle-sized chair."
9. "Hold the circle over the big chair."
10. "Hold the triangle behind the small chair."

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**Concept:** Positional relationships

**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, flannel objects (table, ball, two chairs)

**Activity:** The teacher places the flannel table on the flannel board. A student will then place the ball on top of the table. Someone else may place the ball under the table. The ball may also be placed to the right or left of the table. All objects are then removed from the flannel board. The teacher will place two chairs on the flannel board, thus: . A student put the ball between the chairs. One chair is then removed from the flannel board. The children respond to the following directions:

1. "Place the ball behind the chair."
2. "Place the ball on the chair."
3. "Place the ball under the chair."
4. "Place the ball in front of the chair."

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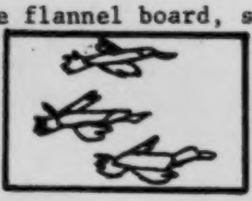
For alternate activities, teacher arranges the objects and requires the children to verbalize the correct response.

**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, three birds made of flannel, three airplanes made of flannel

**Activity:** The three birds are put on the flannel board, similar to the following pattern. It is best to make the birds different colors for easy identification. First, the class should identify the bird that is the highest in the air, and then the one that is the lowest. This activity can be repeated, using airplanes.



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**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, shapes similar to the ones used on the flannel boards, with small magnets attached to the backs

**Activity:** The preceding activity for the flannel board can be used again, this time with a magnetic board and magnetic objects.

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**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, objects used for the flannel board instruction

**Activity:** The activity used for the flannel board is repeated - using the overhead projector.

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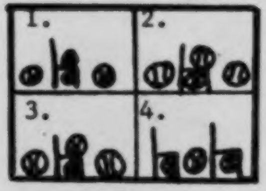
**Concept:** Positional relationships

**Stage of Development:** Semiabstract

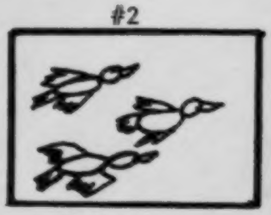
**Equipment:** Overhead projector, pen, prepared transparencies

Examples:

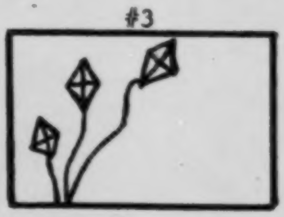
#1



#2



#3



**Activity:** On transparency #1, different children will mark the correct response, with reference to questions concerning the relative position of the ball to the chair(s).

1. "Mark the ball under the chair."
2. "Mark the ball on the chair."
3. "Mark the ball that is behind the chair."
4. "Mark the ball that is in front of the chair."

Using transparencies #2 and #3, the terms "highest" and "lowest" can be discussed.

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## CLASSROOM ACTIVITIES

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|---|---|
| Concept: | Geometry - closed curves |
| Stage of Development: | Concrete |
| Equipment: | None |
| Activity 1: | A large circle is drawn on the floor. One child is asked to stand "on" the circle. Someone else is asked to stand "inside" the circle. A different person stands "outside" the circle. |
| Activity 2: | The children form a circle by joining hands. Someone stands "inside" the circle of children and someone else stands "outside" the circle. |

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|---|---|
| Concept: | Geometry - closed curves |
| Stage of Development: | Semiconcrete |
| Equipment: | Flannel board, flannel animals, pieces of yarn |
| Activity: | The flannel animals are placed on the flannel board. The yarn is placed around some of the animals, and the children are told it is a fence to keep the animals from running away. Someone points to the animals that are "outside" the fence. Someone else points to the animals that are "inside" the fence. |

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|  |  |
|---|---|
| Stage of Development: | Semiconcrete |
| Equipment: | Magnetic board, magnetic objects, chalk |
| Activity: | Several objects are placed on the magnetic board. A closed curve is then drawn around some of the objects. The children are to identify objects "inside" and "outside" the closed curve. |

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**Concept:** Geometry - closed curves

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** The children are told to draw an enclosed curve on their chalkboards. Then, they put two triangles "inside" the curve and three dots "outside" the curve. Their chalkboards should be held up for the teacher's observation.

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**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, overhead projector pen

**Activity:** A closed curve is drawn on the overhead projector. A pupil puts four dots "inside" the closed curve. Another one puts two dots "outside" the closed curve. Still another places two dots "on" the closed curve.

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**Concept:** Positional relationships

**Stage of Development:** Concrete

**Equipment:** Students

**Activity:** Choose two children to assist you for this activity. One child is told to stand in one of the corners of the room while the other is to stand next to the teacher's desk. Someone is then asked to identify the student who is nearest the teacher's desk and who is farthest from the desk. Repeat with other children.

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**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic characters

**Activity:** Draw a house on the magnetic chalkboard. Place several characters on the board in various places. Someone is then asked to identify the character farthest from the house and the one nearest the house. Repeat with other characters.

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Concept: Positional relationships

Stage of Development: Semiabstract

Equipment: Overhead projector, characters

Activity: Repeat the previous activity drawing a house on the overhead and using overhead projector characters.

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# PATTERNING
## (Order and Number Comparisons)

### Introduction

Before, after, and between are important mathematical terms. They can, however, be used in conjunction with non-number activities. The first few pages of this section are concerned with forming patterns (a non-number activity), where these terms can be introduced.

The order of the whole numbers used in counting emerges naturally from work with sets. Beginning with the empty set, one member at a time is added (or joined). The cardinal numbers of the respective sets formed are a natural sequence of the whole numbers, each number being one greater than that in the previous set. Therefore, it is important to work with ordering sets before talking about the ordering of numerals.

Concept: Order - patterns

Stage of Development: Concrete

Equipment: Fruit loops, pieces of string

Activity: The children arrange the cereal pieces on their string in a two, three or four color pattern. The children must form two or more sequences of the pattern.

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Concept: Order - patterns

Stage of Development: Concrete

Equipment: Precut construction paper triangles, circles, squares and rectangles

Activity: The children arrange the geometric shapes in a two, three, or four shape pattern. The children must form two or more sequences of the pattern.

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Concept: Order - patterns

Stage of Development: Concrete

Equipment: Colored blocks, discs, beads, strings for the beads

Activity: The class is divided into three groups, with one group getting blocks, another discs, and the third group getting beads.

Each group works on a pattern of "two" (use two different colored beads, blocks, or discs to form a pattern and then repeat it for two or more sequences). After the children have become proficient at working with a pattern of "two", they should begin working with "three". (Use three different colored objects to form a pattern.) The children can extend their knowledge of patterns to include using four, five, or more different colors and shapes to make sequences.

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Concept: Order - patterns

Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic shapes

Activity: A child is asked to form a pattern of "two" on the magnetic board, using two different shapes.

(e.g., ★○★○★○★○ )

Another child may form a pattern of "three".

(e.g., ★○△★○△★○△ )

The children may also attempt a four or a five pattern.

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Equipment: Flannel board, flannel shapes

Activity: Individual flannel boards may be used, if available. If not, the demonstration flannel board can be used. The children are to form patterns as shown in the previous activity, using flannel shapes.

+ + + + + + + + + + + + + + + +

Concept: Order - patterns

Stage of Development: Semiabstract

Equipment: Overhead projector

Activity: Each child forms a two, three or four symbol pattern on the overhead projector. Some symbol suggestions are: ○ △ □ ● etc.

Use the above activity with individual chalkboards.

+ + + + + + + + + + + + + + + +

Concept: Order - sets

Stage of Development: Concrete

Equipment: Paper plates, boxes, objects to set on the plates

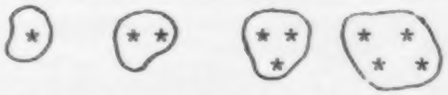
Activity: The students will help the teacher form sets by placing a specified number of objects on the plates. First, they will help form a set having three members, then one with two members, then one with eight members, etc., until each of the ten plates has a set. A representative set for each number from one through ten should be formed. Ten boxes, each labeled with a numeral, should be placed on the teacher's desk in natural order. The numerals will face the students. The teacher then holds up a paper plate so that the objects on it can be seen by all the students. The children then tell into which box the objects must be emptied so that the number of objects matches the numeral on the box. This activity is continued until each set is in the appropriate box (in natural order).

+ + + + + + + + + + + + + + +

Concept: Order - sets

Stage of Development: Semiconcrete

Equipment: Magnetic chalkboard, magnetic cards showing sets 0 - 10

Activity: The children arrange the set cards in natural order. e.g.,  etc.

+ + + + + + + + + + + + + + +

Concept: Order of sets

Stage of Development: Semiabstract

Equipment: Several 3" x 5" cards for each student

Activity: The students draw or paste pictures of sets on the cards, so that each number, one through ten, is represented by a card. Then, they arrange the cards in natural order.

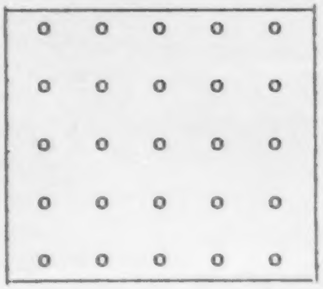
+ + + + + + + + + + + + + + +

**Concept:** Patterning

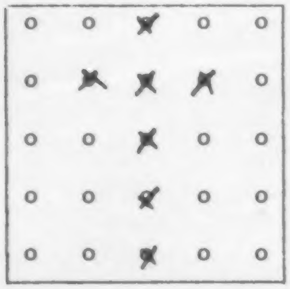
**Stage of Development:** Concrete

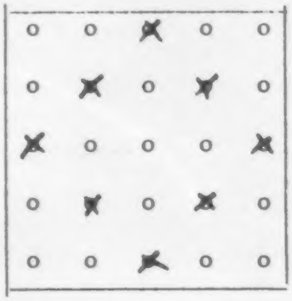
**Equipment:** Pegboard geoboard, similar to the one in the following diagram, which can be constructed:

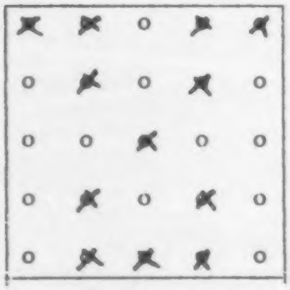
One inch pieces of half inch dowel rods are used for legs. Golf tees or pegs are used to fill the holes. Cut the pegboard so there are five rows of five holes.



Grids matching the holes on the pegboard geoboard are made also. Some examples of the kinds of grid patterns that can be made are illustrated below:







Activity: The child is given a pegboard geoboard and a pattern card, with instructions to reproduce the pattern on the card, using the board and pegs.

After the child has had several opportunities to form patterns that the teacher has prepared on the grids, the child should be required to make up his own pattern. When the child finishes the pattern, ask him to reproduce it on one of the grid cards. He then signs his name on the back of the card and keeps a file of all the cards he makes.

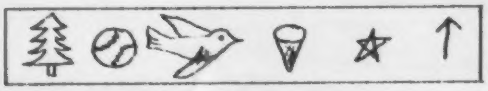
+ + + + + + + + + + + + + + + +

Concept: Patterning

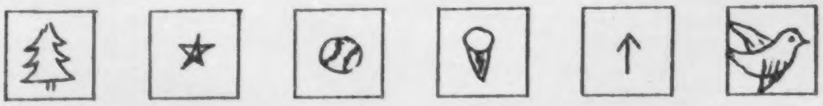
Stage of Development: Concrete

Equipment: Clothesline matching device

Activity: The teacher uses tagboard to make a pattern of objects for the children to copy. For example:



The objects are then placed on individual cards, as follows:



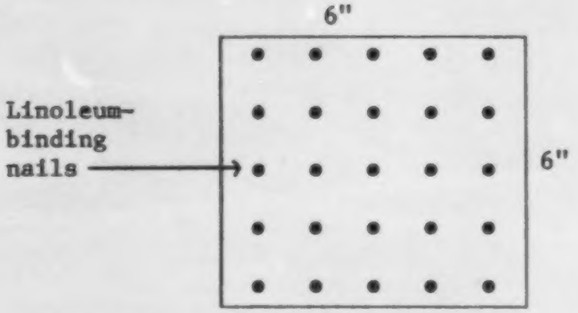
A small magnet is placed on the back of each card. The children use the clothesline matching device, the long tagboard card showing the order of the objects, and the individual cards. The individual cards are used to form a pattern identical to the one on the long card. This is done by placing the individual cards on the wire. The magnet will hold the card to the wire. Several different patterns can be made for the same cards, or new patterns and cards can be made. The terms "before", "after", and "between" can be used with reference to the positions of the different objects.

+ + + + + + + + + + + + + + + +

**Concept:** Patterning

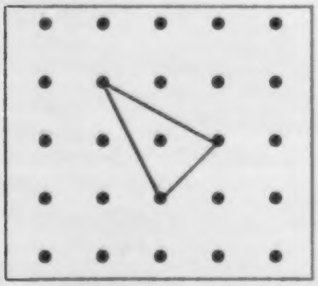
**Stage of Development:** Concrete

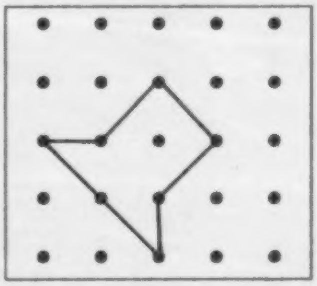
**Equipment:** Geoboards (Instructions for making them follow)

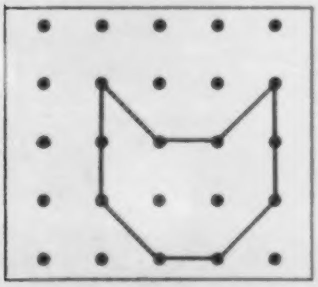


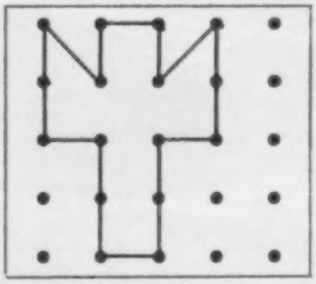
The geoboard can be made out of one half inch plywood, using linoleum binding nails spaced one inch apart. It is a more attractive board if it is painted. The nails are pounded in so that approximately 3/8" of each nail is exposed. Different colored rubber bands should be used on the board.

**Activity #1:** Make a grid on a sheet of paper so there are five rows of five dots separated by one inch. This will match the nails on the geoboard. Make several patterns on these grids that students can copy on their geoboards. Below are some examples of patterns:









The child is given one of the patterns the teacher has made and also a geoboard with a rubber band The child is to use the rubber band to make an identical pattern to the one on the grid.

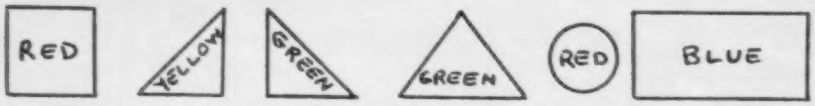
**Activity #2:** An alternate activity would be to give the child a geoboard and a rubber band and ask him to make a design of his own. When he has done this, require him to make a copy of his design on one of the grids the teacher has given him.

+ + + + + + + + + + + + + + +

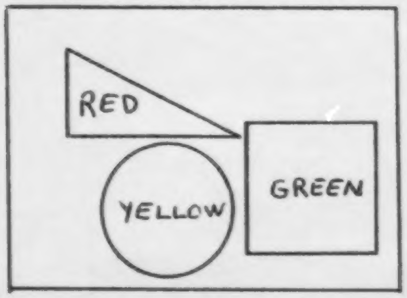
**Concept:** Patterning

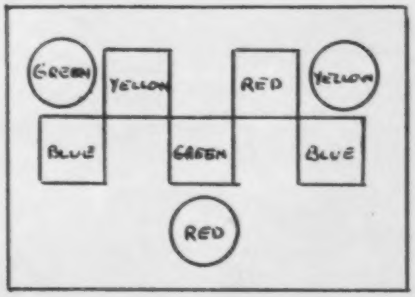
**Stage of Development:** Concrete

**Equipment:** Various flannel cut outs, using different sizes, colors, and shapes, are prepared.



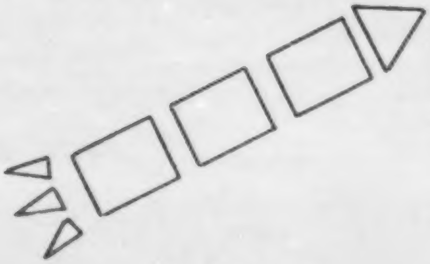
Patterns are then made on 4" x 6" cards. Some examples are given below:

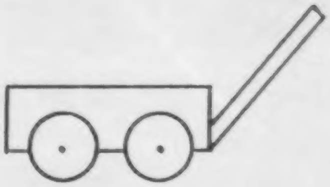




**Activity #1:** The child gets a flannel board and a tray of the flannel characters needed to make the pattern. He is required to duplicate the pattern shown on the card on his flannel board.

Interest can be increased by creating patterns that look like characters (animals, rockets, toys, etc.):





Activity #2: Another activity would be to allow the children to make their own patterns.

+ + + + + + + + + + + + + + +

Concept: Patterning

Stage of Development: Concrete

Equipment: Grid paper

Activity: Have available 1" square grid paper and encourage children to make a two color pattern.

After a few times of this type of activity, children will be encouraged to make three, four and five pattern designs.

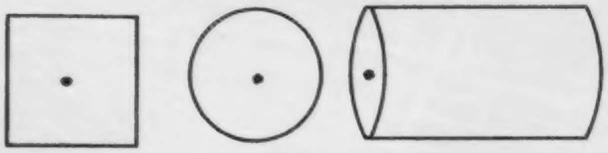
| Red | Blue | Red | Blue | Red |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

+ + + + + + + + + + + + + + +

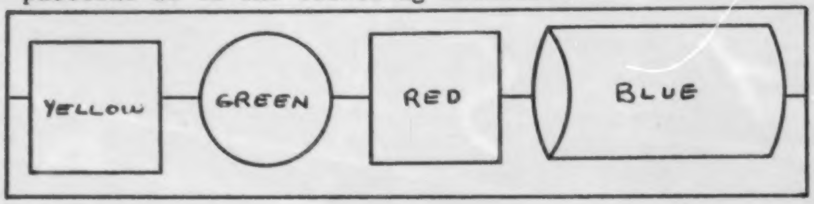
Concept: Patterning

Stage of Development: Concrete

Equipment: String and red, blue, purple, green, orange, and yellow beads of the following shapes:



The teacher prepares 4" x 6" cards, showing patterns as in the following illustration:



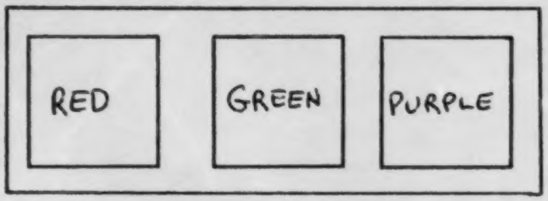
**Activity:** The child gets one of the cards showing a pattern to be reproduced. He also receives a tray full of beads and a piece of string. He then makes the same pattern as seen on the card, using the beads and string he has been given. The pattern should be repeated at least twice. The teacher can then be sure the pupil understands the correct sequence of the beads. An alternate activity would be for the child to make up his own patterns.

+ + + + + + + + + + + + + + + +

**Concept:** Patterning

**Stage of Development:** Concrete

**Equipment:** Blue, red, yellow, orange, green, and purple cubes, 4" x 6" cards showing a pattern as in the following illustration:



**Activity:** The student gets a card showing a pattern to be reproduced, and also a tray full of cubes. He is to make the pattern, placing the cubes end-to-end. The pattern should be repeated so the teacher can be sure the child understands the correct sequence of the cubes.

+ + + + + + + + + + + + + + + +

**Concept:** Patterning

**Stage of Development:** Concrete

**Equipment:** Unifix blocks

**Activity:** Visual R B R B R B . . .

What comes next? Finish this pattern. Make a pattern like it using different colors.

G R R G R . . .

What comes next? Finish this pattern. Make a similar pattern using different colors.

**Auditory** white, green, blue, blue, white, green, blue, blue, . . .
black, black, orange, orange, black, black, orange, . . .

**Number** (Color doesn't matter.)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| R | W | W | B | B | B | G | | | | |

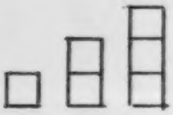
| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y | B | B | B | W | W | W | W | W | O | | | | |

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| O | O | B | B | B | B | Y | Y | Y | Y | Y | Y | G | | | | |

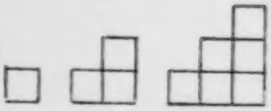
Make trains of these patterns.

1, 3, 6, __, __, __, __,

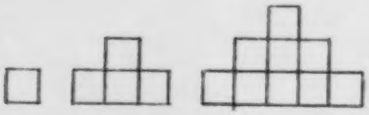
2, 4, 6, __, __, __, __,



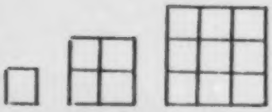
Can you build the next four? What is the pattern?



Continue this pattern. Do you know what it is?



What comes in the next three places?



Build the next three. What is the pattern? Do you know what the tenth one would look like?

Build at least five more patterns of your own. Tell about them.

Using unifix cubes and 1" graph paper make as many patterns as you can for any given number. Color and cut out.

(e.g., Patterns for six -- color doesn't matter)

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | | | | O | | | | | | B | B | B | B | | | | | | | | |
| R | R | | | O | | G | G | | | B | B | | | | | | | R | R | | |
| R | R | R | | O | | G | G | | | | | | | | | | | | R | R | |
| | | | | O | | G | G | | | | | G | G | G | G | G | | | | R | R |
| B | B | B | | O | | | | | | | | | | | | G | | | | | |
| B | B | B | | O | | | Y | Y | Y | Y | Y | Y | | | | | | | | | |

and more . . .

How many are two row patterns?

How many are three row patterns? four row, five row, etc.?

Write number sentences when children are ready.

e.g.,

$1 + 2 + 3 = 6$

Also, have children flip and rotate some of the patterns. Discuss all of your results. What can you discover? Try patterns for other numbers.

Make a long train of this pattern: 4, 2, 4, 2, . . .

Break the train off at different lengths. Continue breaking the train off at the same length each time laying each length side by side. Color your pattern on graph paper. Can you continue coloring the pattern after you have run out of cubes?

e.g.,

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | R | R | R | W | W | R | R | R | R | W | W | R | R | R | R | W | W | R | R | R | R | W | W | R | R |

| | | | |
|---|---|---|---|
| R | R | R | R |
| W | W | R | R |
| R | R | W | W |
| R | R | R | R |
| W | W | R | R |

Make the same long train again. Choose another length and do the same thing. Can you find a pattern?

# COUNTING

## Introduction

RATIONAL: Counting of objects

ROTE: Counting should be deferred until the children are able to establish, with confidence, a number perception with respect to sets of objects (without counting). Some children may have learned to count using a rote, short cut method (being able to recite the number names in order). Children who count by rote need special encouragement to see the relationship between the number name and the number in a set. For example, a child who is learning to count objects ( unlike an adult who counts) should say each number name after he has completed a move, and as he perceives the number of objects. He should arrange the objects that he moves in patterns, so that he and other children can readily perceive the number in successive sets (subsets of the entire set).

The child should be able to count, as follows:

| | |
|---|---|
| Kindergarten | 1 - 20 by ones |
| Grade one | 1 - 100 by ones, twos, fives, tens |
| Grade two | 1 - 100 by ones, twos, fives, tens |
| | 1 -1000 by hundreds |

SERIAL: Rote counting is counting without the aid of objects. Rational counting is the counting of a set of objects. Serial counting is the counting of sensations as distinguished from the counting of objects. The auditory sense will be used in this form of counting. The teacher will make a set of sounds in some manner (hammer on wood, clapping of hands, tapping the table with a pencil, piano, etc.), then the students will state the number of sounds heard.

ORDINALS: Numbers are used not only to tell "how many" but also to indicate a position in an orderly arrangement of objects. Thus, if five objects in a row are counted from one end to the other, objects "one" and "five" are at the ends, and object "three" is in the middle. This is an ordinal use of the number names. Thus the idea of position can be expressed without using the words first, second, third, etc. When one counts a group of objects, the final answer is the same, regardless of the order in which the objects are counted. However, when the number names are used to indicate order or position, the situation is quite different. In a row of five objects arranged in a horizontal line, the object that is "two" from the left will be "four" if the objects are counted from right to left. In this situation, the word "five" is used to tell not only how many objects there are (the cardinal use), but also which object is last in this arrangement (the ordinal or positional use). Children need careful instruction and experience in using number words to tell position. They must become aware that position is relative to the starting point and to the direction in which the "count" is made. The teacher must make certain that the reference point for ordering is at the left, since at this stage of development students are beginning to read (from left to right) and will find it difficult to work from right to left.

## CLASSROOM ACTIVITIES

**Concept:** Rational counting

**Stage of Development:** Concrete

**Equipment:** Children

**Activity:** All boys present should stand. Several different girls may then count the boys. The girls will stand and the boys will count them. This can be a daily activity.

*******************************

**Stage of Development:** Concrete

**Equipment:** Counters (discs, blocks, buttons, sticks, etc.)

**Activity:** Each child has a set to count. The number of elements in this set will vary according to the maturity of the child. The children are asked to count the number of elements in the set. They should know that when they count, the number stated represents the cardinal number of the set of objects counted thus far. Several lessons should be carried out, using different counters and varying the number of elements in the sets.

*******************************

**Concept:** Rational counting

**Stage of Development:** Concrete

**Equipment:** Paper bags, objects to place in the bags

**Activity:** The students are given paper bags containing six through ten objects. They are to determine the number of items in their bags and are not to tell anyone the number. One student then comes to the front of the class, shakes his bag, and asks the class to guess the number of objects it contains (by giving hints of "more than" or "fewer than" after each guess). The student who guesses correctly wins. He may then count the objects as he takes them, one by one, out of the bag; he then lets the class try to guess the number of objects in his bag.

*******************************

**Stage of Development:** Concrete

**Equipment:** Paper bags, objects to place in the bags, numeral cards

**Activity:** Each student receives a bag with one through ten similar objects (e.g., discs). Each one then counts the objects in his own bag. The teacher holds up a numeral card and asks which students have as many objects as are represented by the numeral card. Those children who think they do will empty their bags and, one at a time, count the objects and report the results to the class.

*******************************

**Concept:** Rational counting

**Stage of Development:** Semiconcrete

**Equipment:** Individual flannel boards, flannel objects in an envelope

**Activity:** The number of objects that are placed in the envelopes will depend upon the children's maturity of rational counting. Each child must count the objects in his envelope. When the child counts the objects, the teacher must make sure he understands that when he states a number, this number represents the cardinal number of the set of objects counted thus far.

*******************************

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic boards, magnetic shapes

**Activity:** A set of magnetic shapes is placed on the magnetic board. One child counts the shapes on the board. When the child counts, he must be required to move each figure as he counts it. The number stated in the counting is the cardinal number of the set.

*******************************

**Concept:** Rational counting

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, coins, pop bottle lids, dog biscuits, jacks, etc.

**Activity:** A set of objects is placed on the overhead projector. One pupil comes forward and counts the objects. When the child counts the objects, he must move each object as he counts it. The number stated, when counting the objects, represents the cardinal number of the set that is being counted.

*******************************

Stage of Development: Semiabstract

Equipment: Cards showing sets (1-20 elements)

Activity: Each child is given one of the cards. The teacher then has an option of several activities:

1. The teacher will say a number and all the children having that many elements in their sets will stand up. The teacher then checks to see if they are correct.
2. Each child stands up, one at a time, and counts (aloud) the elements in his set, stating the cardinal number of the set counted.
3. A card showing a numeral is flashed to the class. All the children having sets where the cardinal number is this numeral should stand up. The teacher checks those who are standing for correctness.

This procedure may be repeated by collecting the cards, mixing them up, and redistributing them.

********************************

Concept: Rational counting - beginning with a certain number

Stage of Development: Concrete

Equipment: Sets of seven through ten objects for each student

Activity: Each student receives a set of seven through ten objects. He is asked to arrange them on his desk so that members of the set do not touch. The students group three members of the set (without moving any of the objects) and count the rest of the set. Then, four objects are grouped, and the rest of the set is counted, beginning with the number four. This process is continued until the students realize that the objects in the smaller group (subset) do not have to be counted, and that they can begin to count with the number associated with the subset.

********************************

NOTE: For rational counting by twos and fives, use the same objects and activities you used for counting by ones.

## CLASSROOM ACTIVITIES

Concept: Serial counting to ten

Stage of Development: Abstract

Equipment: Ball

Activity: The ball is bounced a predetermined number of times (out of the view of the class). One student tells how many times the ball was bounced. This can be repeated by bouncing the ball a different number of times.

*******************************

Stage of Development: Abstract

Equipment: None

Activity: The children will close their eyes. The teacher claps his hands together a predetermined number of times. The children then open their eyes and someone tells how many times the teacher clapped his hands. Other methods for making sounds can be used: stomping the foot, tapping the desk, etc.

*******************************

Stage of Development: Abstract

Equipment: Hammer, block of wood

Activity: The children close their eyes while the teacher hammers the block of wood. They must then tell how many times the hammer strikes the block of wood.

*******************************

Stage of Development: Abstract

Equipment: Metal triangle, metal rod

Activity: The children close their eyes, while the teacher strikes the triangle several times. They must then tell how many times the triangle was struck.

*******************************

Stage of Development: Abstract

Equipment: Bells

Activity: The children close their eyes while the teacher rings the bells. They must tell how many times the bells were rung.

*******************************

Concept: Serial counting

Stage of Development Concrete

Equipment: Tactile box, wooden blocks

Activity: Place up to ten blocks in the tactile box. The children place their hands inside the box to determine the number of objects by using their sense of touch.

*******************************

## CLASSROOM ACTIVITIES

Concept: Ordinals

Stage of Development: Concrete

Equipment: Children

Activity: The class is divided into the set of boys and the set of girls. The boys are lined up in a row, while the girls are asked questions about the boys, relative to their positions. Some of the questions are listed below:

> Which boy is last?
> Which boy is first?
> Who is in the fourth position?
> Who is second? etc.

The boys then sit down and the girls form a row, while the boys answer questions about the girls.

******************************

Stage of Development: Concrete

Equipment: Toy cars

Activity: The cars are arranged as if they're in a race. The teacher points to different cars, questioning the children about their positions. The children should also be asked questions where they have to identify cars that occupy certain positions.

******************************

Stage of Development: Concrete

Equipment: Manipulative objects

Activity: The objects are arranged in a row. The class then talks about the positions that the objects occupy (first, second, third, etc.). The children identify objects that are in certain positions.

******************************

Stage of Development: Concrete

Equipment: None

Activity: Five students are arranged in a row, facing the class. The rest of the class must determine who is first, fifth, last; who is third from the window, from the teacher's desk, etc. Different reference points can be used to order the group (e.g., the wall, chalkboard, bookcase, a specific desk). The five students are asked to walk toward the door, and the rest of the class determines who will reach the door first, fifth, and last. (The teacher should emphasize that the last member of the group bears the number of the set.) The class may also arrange the group, using various reference points.

*******************************

Stage of Development: Concrete

Equipment: Chairs

Activity: Several chairs are placed in front of the children. The teacher and class then talk about the position that each chair occupies. The chairs can even be marked with the numerals 1, 2, 3, 4, 5, etc. Different children are asked to take certain chairs. (E.g., Mark, sit on the third chair.") Other directions may then be given:

1. "Will the child in the fifth chair please stand up?"
2. "The child in the sixth chair will return to his desk."

*******************************

Concept: Ordinals

Stage of Development: Semiconcrete

Equipment: Flannel board, several felt birds of different colors

Activity: The birds are placed on the flannel board as if they were flying in a race. The teacher points to different birds and questions the children as to their relative positions. Questions similar to the following may be asked:

1. "In what position is the red bird?"
2. "In what position is the green bird?"
3. "What color is the bird in the third position?"
4. "What color is the bird in the sixth position?"

*******************************

Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic objects

Activity: The objects are arranged in a row so it is obvious which one is first. The teacher picks up different objects and asks the class about their relative positions. A child may also go to the board and pick up objects named by the teacher (with reference to their positions).

*******************************

Concept: Ordinals

Stage of Development: Semiabstract

Equipment: Overhead projector, objects, or pennies

Activity: The objects, or pennies, are arranged in a row on the overhead projector. The teacher must designate which object is first and in what direction the children will count. Such questions as the following may be asked:

1. "Which object is in the third position?"
2. "Which object is last?"
3. "In what position is this object?"

* * *

Stage of Development: Semiabstract

Equipment: Overhead projector, transparencies of objects in a row or in a race

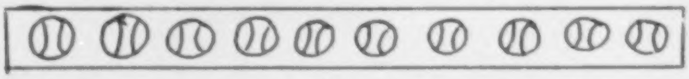
Activity: The transparencies are placed on the overhead projector. Using an overhead projector pen, one of the children will circle the object occupying a certain position (e.g., the third, the fourth, etc.); or the teacher may circle one of the objects and have the children reply as to its position.

* * *

Stage of Development: Semiabstract

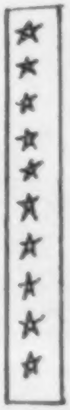
Equipment: Transparencies showing objects arranged in either a vertical or a horizontal row

Activity: The transparencies are placed on the overhead projector. See illustration below.



The teacher then asks the following questions?

1. Which ball comes first?
2. Which is the seventh ball from the left?
3. Which is the ninth star from the top?
4. Which is the fourth star from the top?
5. Which star follows the seventh star from the top?
6. Which star is the bottom star?



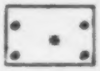
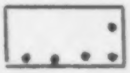
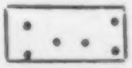
* * *

NOTE: The same activities can be used for ordinal counting using different numbers of objects (1-20).

## SET RECOGNITION

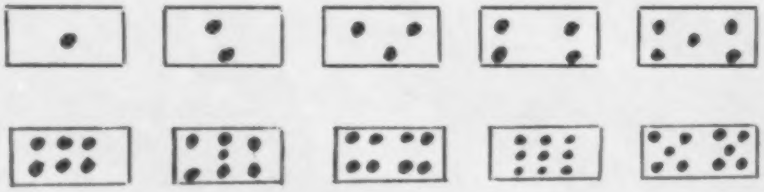
### Introduction

One-to-one matching is the basis for the mental process through which a child learns the cardinal numbers of sets. For example, he may see a set of three objects, but not be aware of the number for this set. The next time he sees a set containing this number of elements, he matches (consciously at first, then later unconsciously) this set with the set whose number he knows, and concludes that the new set also contains three elements.

The child should learn to recognize the number of elements in a set, as a group, without counting. In the beginning, some children may need to count the elements, but there should be constant encouragement to recognize the number of a set without counting. By arranging the elements in easily recognizable patterns, the children can be helped to recognize sets without counting. For example, it is easier to recognize five  from this pattern, than from this one . Most people would recognize the following pattern  as a single set of six, but some may see it as sets of four and two, while others will see two sets of three. Seeing a set of six in two parts (subsets) is a natural preparation for the addition equations, $4 + 2 = 6$ and $3 + 3 = 6$.

The child should learn to recognize immediately, without counting, groups of two, three, four, and five objects. The child's understanding of these small groups will enable him later to break up and identify larger groups. Moreover, research and experience have shown that children understand arithmetic better, and can reach high levels of achievement, when they are taught to recognize and use groups.

The basic configurations used for one through ten objects will be as follows:



## CLASSROOM ACTIVITIES

Concept: Empty set

Stage of Development: Concrete

Equipment: Children, chairs

Activity: Four boys are placed in four chairs in front of the class. The class is then asked to describe the set of boys. Two boys are told to go back to their seats. The class now describes the new set of boys. The remaining two boys are told to take their seats. The class then describes the set of boys remaining.

******************************

Stage of Development: Concrete

Equipment: Dish, blocks

Activity: A dish which contains five blocks is placed in front of the class. The teacher asks the class to tell how many blocks the dish contains. Two of these blocks are then removed. The children describe the new set of blocks. The last three blocks are removed from the dish, and the children describe the set of blocks that now remains.

******************************

Stage of Development: Concrete

Equipment: A container with several counters in it

Activity: Objects are removed, one by one, from the container. Each time this is done, the students should feel the inside of the container to see if there are any objects left. They are then asked to describe the set that is in the container. At the end of this sequence of activity, the last remaining object is removed. The students are allowed to place their hands in the container to "feel" the absence of members. They are told, "This is a set with no members. It is the empty set."

******************************

Stage of Development: Concrete

Equipment: Assorted objects (e.g., bottle caps, blocks, discs, sticks, etc.)

Activity: The students are asked to construct a set of all the bottle caps on their desks. Then they will construct a set of all the blocks on their desks. This process is continued for three or four types of sets. The students are then asked to construct the set of elephants on their desks. (The set of elephants is an example of the empty set.) The teacher should say, "Since there are no elephants on your desks, there can't be anything in the set of elephants on your desks. This is an empty set."

******************************

Stage of Development: Concrete

Equipment: None

Activity: Each child who belongs to the set of all the children in the classroom is asked to stand. Then the teacher asks that only students with black hair remain standing; next, he asks only those who are girls with black hair to remain standing. This process is continued until no students are left. Finally, the teacher asks which members are in a certain specified set. (E.g., "Who belongs in the set of students with black hair? Who belongs in the set of girls with pink ribbons in their hair? Who belongs in the set of people with three ears?" The ears of several students can be counted to indicate the process used to determine whether there are members of this set. Since there are no students in this set, it is the empty set.)

******************************

Concept: Empty set

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel board stars, squares, ducks, and moons (two of each object is sufficient)

Activity: The class is asked to construct a certain set (e.g., a set of ducks). A series of flannel board objects is held up so that the class can determine whether the object belongs in the set being constructed. After four or five sets are constructed in this manner, the class should be asked to construct the set of dogs. One object at a time is held up and the children are asked whether or not the object should go into the set. When all the objects have been viewed, the teacher asks the students, "What do we call the set of dogs?" Hopefully, the answer will be, "The empty set, since there are no members in the set of dogs on the flannel board."

******************************

Concept: Empty set

Equipment: None

Activity: The children are asked to determine which of the following are examples of the empty set:

1. The set of girls in the classroom who have red hair.
2. The set of pupils in the class who are over forty years old.
3. The set of girls in the class.
4. The set of boys in the class.
5. The set of people in the classroom who are over twenty years old.
6. The set of people in the school who have green hair.
7. The set of children with two heads.
8. The set of children in the class.

******************************

Concept: Empty set

Stage of Development: Semiabstract

Equipment: Individual or demonstration chalkboards

Activity: The students are to draw a set of four objects on their chalkboards. One child is asked to give the cardinal number of this set. The students then erase one object at a time, naming the new cardinal number each time. When there are no objects remaining in the set, the teacher asks, "What kind of set do we now have?" Someone may describe the cardinal number of this set.

******************************

Equipment: Overhead projector, objects

Activity: This activity is similar to the preceding one. One object at a time is removed until none remain. The children should describe the set each time one member is removed, until the last response becomes, "The empty set."

******************************

# CLASSROOM ACTIVITIES

Concept: Primary group recognition

Stage of Development: Concrete

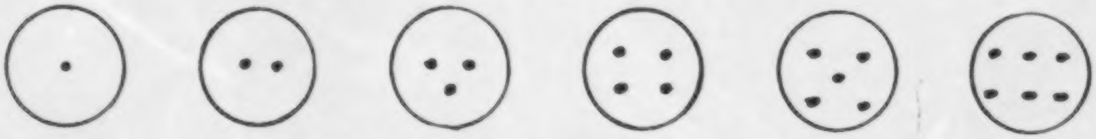
Equipment: Blocks, bottles, sticks, etc.

Activity: Sets of two, three, four, and five objects are shown to the children. They should be able to state the cardinal number of the set without counting. Several different objects can be used to form the sets, so the children will have the advantage of different experiences.

*******************************

Stage of Development: Concrete

Equipment: A large circle drawn on the floor, circles cut from construction paper (showing the sets, one through six)



Activity: The circles containing the sets are placed inside the large circle to act as stepping stones, while the large circle represents a pond. Each child tries to walk across the pond on the stepping stones. He may step on the next stone, if he can identify the cardinal number of the set on it. If he misses the number, he falls in the pond and has to return to the bank.

*******************************

Concept: Set recognition

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel objects, sheet of construction paper

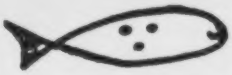
Activity: The teacher forms sets of one, two, three, four, and five objects on the flannel board, shielding them from the view of the class by using the sheet of construction paper. The sets formed should be in the following pattern:

• , ⁚ , ⁖ , ∷ , ⁙ .

The sheet of paper is raised just long enough to give the class time to visualize the set, but not long enough for the children to count the members. The teacher then questions the students as to how many members they observed. This activity can be repeated with the magnetic board and objects.

******************************

| | |
|---|---|
| Concept: | Set recognition |
| Stage of Development: | Semiabstract |
| Equipment: | Several fish cut out of construction paper (with one of the basic configurations on each one, and a paper clip for its mouth), pole, string with small magnet on the end |



Activity: A large circle, that will serve as a pond, is drawn on the floor. The fish are placed in the pond. Using the pole, with the magnet acting as a hook, the children are able to catch the fish by bringing the magnet into contact with the paper clip. They must identify the cardinal number of the set without counting. This activity can be repeated many times.

******************************

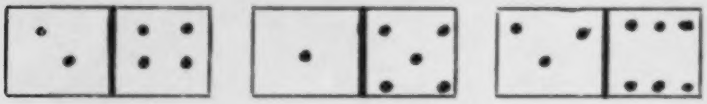
| | |
|---|---|
| Stage of Development: | Semiabstract |
| Equipment: | Overhead projector, various shapes to show images on the screen (e.g., pennies, rabbits, pumkins, snowmen, etc.), sheet of construction paper |

Activity: One of the basic patterns ( • , ⁚ , ⁖ , ∷ , ⁙ ) is formed on the overhead projector, and the image is blocked with a sheet of construction paper. The sheet of paper is raised just long enough to give the class time to visualize the set but not long enough to count the members. The children then tell the number of members they observed in each pattern. This activity is also suitable for use on the chalkboard.

* * *

**Stage of Development:** Semiabstract

**Equipment:** Flash cards showing patterns similar to dominos:

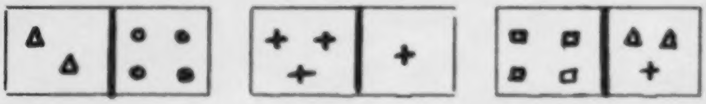


**Activity:** One half of the card is covered by the teacher's hand, the other half is exposed to the children. The children are required to identify the number of dots on the card without counting. The other side of the card can then be covered, while exposing the one previously covered. This set should also be identified.

* * *

**Stage of Development:** Semiabstract

**Equipment:** Set dominos:



These can be made from tagboard

**Activity:** The teacher covers one end of the domino and exposes the other end to the class. The children are not to be given time to count the elements in the set, as it is important that they be able to recognize the number of elements in the set by sight. The teacher then asks the question, "How many elements are in this set?"

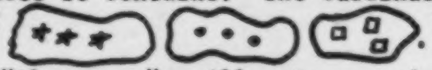
* * *

NOTE: Give your students enough opportunities with these activities so they can recognize all the set patterns 0-10 at sight. This will be important when they study joining sets, separating sets, equivalent sets and partitioning sets.

## ONE TO ONE MATCHING

### Introduction

One of the most basic concepts in mathematics is that of a set. Sets and set relationships form an integral part of the entire range of mathematical development. Intuitively, a set is simply a group, or collection of objects. Any collection of objects that can be placed together, treated together, or thought of together can be considered as a set. The objects that form a set are called its elements or members. The essential characteristic of a set is that it be well-defined. This means that it should be possible to tell which objects are in the set.

The cardinal number of a set is the property of the set which tells how many objects it contains. The cardinal number of each of the following sets is three:  The property these sets have in common is their "threeness". All sets containing three objects belong to the equivalence class named by the cardinal number, 3. No one can show all the sets in an equivalence class. Therefore, any set in the equivalence class can be chosen as a representative of the entire class, since it is typical with regard to number property, and may be matched one-to-one with any set in its class.

## EQUIVALENT SETS

This concept is of great importance because it is the basis of showing that sets with the same number of elements (equivalent sets) can be designated by the same numeral. Students can determine whether or not two sets are equivalent by developing a scheme for pairing up the elements of the sets. If the elements of the two sets can be paired, with no remaining elements in either set, then the sets are equivalent.

When two sets are paired in this manner, it is also true that a one-to-one correspondence exists between the two sets. Therefore, two sets that can be placed in a one-to-one correspondence are equivalent. The order in which objects of two sets are matched is not important.

The phrases "as many as", "equal to", and "same number" may be introduced at this stage of mathematical learning, instead of using the term equivalent. When matching sets, connecting lines should be short. Students often have difficulty following or tracing a long path.

## CLASSROOM ACTIVITIES

Concept: One-to-one matching - equivalent sets

Stage of Development: Concrete

Equipment: Several chairs

Activity: A set of chairs is placed in front of the children. One of the students then forms a set of children equal in number to the set of chairs. In order to check the student's accuracy, the children should sit in the chairs. If all the chairs are filled, with none left over, then the sets are equivalent.

This may be repeated with different sets of chairs.

*******************************

Stage of Development: Concrete

Equipment: Counters (blocks, sticks, discs, etc.)

Activity: The teacher forms a set of blocks (counters) that has the same cardinal number as there are children present in the class. To determine if there are the same number of blocks as there are children, each child must pick up a block. If all blocks are picked up, and every child has one, then the set of children and the set of counters are equivalent.

This activity can be repeated for a set of girls and a set of boys.

*******************************

Concept: One-to-one matching - comparisons - equivalent sets

Stage of Development: Concrete

Equipment: Counters (blocks, sticks, discs, bottle caps, etc.)

Activity: The teacher forms two sets of counters that contain the same number of elements. He then discusses with the children the ways of determining whether they contain the same number of elements. (Some children will suggest counting. Hopefully, someone will suggest some form of matching.) A matching scheme should then be used to find out if the sets are equivalent. (They are if every counter has a partner.)

*******************************

**Concept:** Comparison - equivalent sets

**Stage of Development:** Concrete

**Equipment:** A set of blocks for each student, a set of pencils

**Activity:** The students are to form sets of the blocks and pencils that are given to them. The children are then asked whether they have as many blocks as they have pencils. The teacher also asks them how they might show that there are more or less pencils than blocks. If they can't suggest a method, the instructor should show them that one way to find the answer is to pair each block with a pencil (so that each block and each pencil is used only one time). When the students have done this, the teacher asks again, "Are there as many pencils as blocks?"

*******************************

**Stage of Development:** Semiconcrete

**Equipment:** Set dominos

**Activity:** Set dominos are dominos which contain sets on each end. There are 36 dominos which provide several alternate grouping patterns for each number, zero through five. The dominos are passed out, a few to each child. The dominos are placed face up, when working with younger children. An older child may wish to keep them face down, so others cannot see them. One child is chosen to be first. He lays his domino in the center of the group. Then each child, in turn, tries to make a match at either of the ends. Only two ends are "going" at any one time. This is continued until all dominos have been played or the play is stalemated.
(Note: If no set dominos are available, regular dominos can be used.)

*******************************

**Concept:** One-to-one matching - equivalent sets

**Stage of Development:** Semiconcrete

**Equipment:** Individual flannel boards, flannel objects

**Activity:** Each child is given an even number of objects. The objects are then separated into two equivalent sets. The equivalence should be determined in the following manner: The teacher says to the children, "Using your hands, remove an element from each set. Continue removing one element at a time until all elements are gone from one or both sets. If all the elements are gone from both sets, the sets are equal to each other."

******************************

Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic shapes

Activity: Two sets are formed on the magnetic board. The students can then see if they have the same number of members by drawing lines between the sets (connecting the members one-to-one). If no members are left unpaired, the sets are equivalent.

******************************

Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic shapes

Activity: The teacher makes a set on one side of the magnetic board, using magnetic shapes. A child is chosen to form a set on the other side of the board, equivalent to the one formed by the teacher. Another child will determine the equivalence by drawing one-to-one lines to pair the elements of the sets.

******************************

Concept: One-to-one matching - equivalent sets

Stage of Development: Semiabstract

Equipment: Overhead projector, flannel shapes, pennies, etc.

Activity: Two sets are formed on the overhead projector. It can be determined whether they have the same number of elements by pairing them, using an overhead projector pen to draw lines from elements of one set to elements of the other set. If each element has a partner, the sets are equivalent.

******************************

Stage of Development: Semiabstract

Equipment: Overhead projector, flannel shapes, pennies, etc.

Activity: Two sets are again formed on the overhead projector. The children can see whether they have the same number of elements as the teacher removes objects from the sets (one at a time from each set). If all elements are removed, the two sets are equivalent.

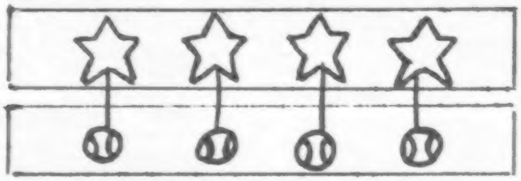
This activity should be repeated with different sets; some equivalent, some not.

Concept: Comparisons - equivalent sets

Stage of Development: Semiabstract

Equipment: Chalkboard

Activity: Two sets, with the same number of members, are drawn on the chalkboard. A student then comes to the board and pairs the members of one set with the members of the other. He may draw a line to indicate the pairings. For example:



The teacher points to one set and asks, "Has this set more members than the other set? Has it fewer members? Has it exactly as many members?"

******************************

Concept: One-to-one matching - equivalent sets

Stage of Development: Semiabstract

Equipment: Chalkboard or individual chalkboard

Activity: The children (either at the chalkboard or on individual chalkboards) form two sets that contain the same number of elements. Equivalence is determined by pairing the elements of the two sets, one-to-one.

******************************

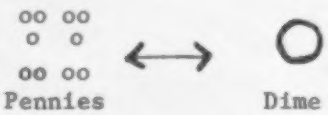
## NON-EQUIVALENT SETS

The concept of non-equivalent sets stems from the concept of equivalent sets. Equivalent sets are sets that can be placed in a one-to-one correspondence with each other. They always have the same number of elements. Non-equivalent sets, therefore, are sets that cannot be placed in a one-to-one correspondence and do not have the same cardinal number. In order to show that some sets are non-equivalent, a matching scheme must be developed which will result in certain members of a set being left unpaired with members of another set. Terms such as "too many", "left over", "more than", "fewer", "less than", and "too few", may be used in a discussion of the results of the pairings.

THE SAME TEACHING STRATEGIES CAN BE USED FOR NON-EQUIVALENT SETS AS FOR EQUIVALENT SETS, USING TWO SETS CONTAINING A DIFFERENT NUMBER OF ELEMENTS.

| | |
|---|---|
| Concept: | Comparison - One to many matching |
| Stage of Development: | Concrete |
| Equipment: | Coins, one dollar bill, ten dollar bill |
| Activity: | The concept of one to many matching is important in learning the value of the various pieces of money. To demonstrate the concept of one to ten matching, the teacher should require the students to pair pennies and dimes so they have the same value. |

e.g.



Similarly, pennies and a half dollar would yield 50 pennies per 1 half dollar; pennies and a dollar bill, 100 pennies per dollar bill; pennies and a ten dollar bill, 1000 pennies per ten dollar bill. The many to one matching concept is also valuable when studying place value and metric measurement.

# RECOGNITION OF NUMERALS

## Introduction

### NUMBER VERSUS NUMERAL

Keeping in mind that the concept of number is an abstraction, let us now turn to the numerals which name these numbers. A numeral is a symbol (such as the character "5" or the word "five") for a number. The number is an idea for which the numeral stands.

It is difficult to maintain the correct use of the words "number" and "numeral". Occasions will arise when it will not be clear whether one should say number or numeral. Also, there will be times when correct usage will be awkward. A child may be asked to go to the chalkboard to write a 2, and the teacher might say, "Go to the board and write the number two." This is incorrect usage, but it would be clumsy to say, "Go to the board and write the numeral for the number two." One should not get bogged down in language. Children understand that "write the number two" refers to the symbol for the number two.

Here is a good rule for the use of the number-numeral terminology. Whenever a person is referring to the symbol itself and not to the number, he should say numeral; in all other cases, say number. Keep in mind, also, that it is better to abuse the language slightly than to make a major issue of it and confuse the children.

## CLASSROOM ACTIVITIES

**Concept:** Numeral recognition

**Stage of Development:** Concrete

**Equipment:** Plastic bottles numbered 1-10, small rubber ball

**Activity:** The bottles are lined up in a row. The ball is rolled by one of the children with the purpose of knocking down one or more of the bottles. The child must identify the numerals on the bottles knocked down.

******************************

**Stage of Development:** Concrete

**Equipment:** Large circles drawn on the floor, the numerals 5-10 drawn inside these circles, beanbag, counters, (blocks, discs, etc.)

**Activity:** The children line up in two rows. The first two children are given bean bags. The beanbags are to be thrown with an effort to get them inside the circles. If a bag lands in a circle, the child is required to tell what numeral is inside that circle. If he is correct, he is given a counting object. He then goes to the end of the line. The child with the most counting objects at the end of the game is declared the winner.

******************************

**Stage of Development:** Semiconcrete

**Equipment:** Large circle on the floor, construction paper stepping stones showing numerals

**Activity:** The teacher explains to the class that the interior of the circle is a pond. The stepping stones are arranged inside the pond. The child crosses the pond by stepping on the stones. He must be able to identify the numeral on a stone before he is allowed to step on it. If he cannot, he is told that he has fallen in the pond and must go back to the bank.

******************************

**Stage of Development:** Semiconcrete

**Equipment:** Fish made from construction paper (with paper clips for mouths), small fishing pole with a small magnet for a hook, numerals on the sides of the fish

**Activity:** A circle representing a pond is drawn on the floor. The fish are placed in the pond. Three or four poles may then be used at the same time. One of the children can catch a fish by bringing the magnet into contact with the paper clip used for the fish's mouth. The child must declare how many pounds the fish weighs by reading the numeral on its side. All the children should have an opportunity to catch a fish.

******************************

Stage of Development: Semiconcrete

Equipment: Large number line on floor

Activity: The number line is taped to the floor. The teacher then stands beside one of the numerals and questions the children as to its identity. This is repeated for different numerals. The children may also be asked to stand on a particular numeral.

******************************

Stage of Development: Semiconcrete

Equipment: Hundreds board, small disc with pin

Activity: The hundreds board is placed so all can see it. The children are told to close their eyes. One of the numbers is covered with the small disc. The children then open their eyes and tell the teacher what numeral has been covered.

******************************

Stage of Development: Semiconcrete

Equipment: Large, styrofoam, stand-up numerals

Activity: The styrofoam numerals are arranged in order, and then the following activities may occur:

1. The teacher points to different numerals and requires the children to identify each one.
2. Some child is asked to come forward and pick out a certain numeral.
3. One of the numerals is knocked over by the teacher, who then states that this numeral got tired and decided to take a nap. Someone tells which numeral decided to take a nap.

******************************

Concept: Numeral recognition

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel numerals, sheet of construction paper

Activity: A numeral is placed on the flannel board. It is shielded from the view of the children by the use of the construction paper. The teacher raises and lowers the paper quickly, giving the child just enough time to get a good visual image of the numeral. He is then required to identify the numeral behind the paper. This activity can also be carried out with the magnetic board and magnetic numerals.

*******************************

Concept: Numeral recognition

Stage of Development: Semiabstract

Equipment: Overhead projector, small transparencies showing numerals, sheet of construction paper

Activity: One of the transparencies is placed on the overhead projector. It is kept covered by the sheet of construction paper. The teacher raises and lowers the paper quickly, giving the child enough time to get a good visual image of the numeral. He must then identify the numeral that was exposed to the class.

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Stage of Development: Semiabstract

Equipment: Chalkboard

Activity: A numeral is written on the chalkboard. The teacher shields the class with his body. He steps away from the numeral and then covers it again. The children are asked to identify the numeral on the board.

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Stage of Development: Semiabstract

Equipment: Tree branch in a stand (both painted white), construction paper circles, triangles, and squares with numerals on them

Activity 1: Geometric shapes showing numerals are placed on the tree branch by means of paper clip hooks. Different children then go to the tree and take off a decoration showing a certain designated numeral.

Activity 2: The geometric shapes are passed out to the children. All the children who have a certain designated numeral will come forward and place it on the tree. This is continued until all the children have placed their numerals on the tree.

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Stage of Development: Semiabstract

Equipment: "Show-me-boards", cards with numerals on them

Activity: The children place the cards on their desks with the numerals face up. They are asked to show the teacher a certain numeral (e.g., 7). They will do this by selecting the numeral and placing it on the "show-me-board". They then hold the board up for the teacher's inspection. Children who hold up the wrong numerals may need individual assistance.

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Concept: Numeral recognition

Stage of Development: Semiabstract

Equipment: Hundreds board, catchers mitt

Activity: The teacher "catches" a certain number, while a child identifies it.

A child used the catchers mitt to "catch" a number designated by the teacher.

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Stage of Development: Semiabstract

Equipment: Numeral cards

Activity: Hand out numeral cards to all the children in the class. State one of the numerals verbally. Require the child having this numeral to stand and show it to the rest of the class.

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Stage of Development: Semiabstract

Equipment: Numeral cards, pocket chart

Activity: Hand out randomly sequentially numbered numeral cards to all the children in the class. Place the first number of the sequence in the pocket chart. Ask the children to identify the next numeral of the sequence. Require the child having it to place it after the numeral which is in the chart.

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NOTE: The size of the numeral studied should depend upon the ability of your children. Use the same activities, merely change the numerals.

# NUMBER WORDS

## Introduction

If a child can recognize and use the number words one, two, three, four, five, six, seven, eight, nine, ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, and one hundred, he should be able to read any number word from one to nine hundred ninety-nine. Being able to read number words, and to write the numeral that represents the number word, is essential in the writing of checks. This section will concentrate on number word recognition and on the numeral associated with the number word.

## CLASSROOM ACTIVITIES

Concept: Number words

Stage of Development: Concrete

Equipment: Miscellaneous objects, box, number word cards

Activity: The objects are shown to the students. They are told that the teacher is going to place several of the objects in the box, and that they are then going to be asked to tell how many objects are in the box. The objects should be transferred, one at a time, from the teacher's desk to the box, making certain that each object is visible only as it is transferred. The students count the objects as they are placed in the box. One child may then show the teacher the number word that describes the set.

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**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, flannel objects, numerals, number words

**Activity 1:** Sets of flannel objects are put on the flannel board. A student is asked to name the number of the set. As he names the number he must place both the numeral and the number word beside the set. The process should be continued until the students have had some work with each number word from zero to ten.

**Activity 2:** A number word is placed on the flannel board. The students form a set for this number word on their individual flannel boards.

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**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, twenty-one flannel objects, flannel board number words from one to twenty-one

**Activity:** Twenty-one objects are placed on the flannel board, and the twenty-one number words are arranged, in order, across its top. The teacher points to one of the number words (e.g., six) and asks a student to come to the board and count out the specified number of objects. He should also form a set that is representative of the specified number word, next to that word. This process can be continued with other numbers and other students. It is important that the student counts the objects for the set.

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**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic cards showing number words, "show me-boards", cards showing numerals

**Activity:** A number word is placed on the magnetic board (e.g., nine). Someone is asked to read it and to show the numeral that it represents, using his "show-me-board".

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**Stage Of Development:** Semiabstract

**Equipment:** Chalkboards, number word cards

**Activity:** Hold up a number word card for all to see. Require the children to write the numeral representing this number word on their chalkboards.

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**Stage of Development:** Semiabstract

**Equipment:** Number word cards, pocket chart

**Activity:** Require a child to arrange a given number of these number word cards in the pocket chart in their proper sequence.

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**Stage of Development:** Semiabstract

**Equipment:** Number word cards, pocket chart

**Activity:** Place several number word cards in the pocket chart. Choose two students of equal ability to stand behind a line drawn about 10 feet from the pocket chart. State verbally one of the number words in the chart. The first student to walk to and pick up the correct number word is the winner. Repeat with other pairs of students.

At the end of the class period you could have the winners compete to find a grand winner.

******************************

**Stage of Development:** Semiabstract

**Equipment:** Cards showing number words and numerals representing the same numbers.

**Activity #1:** Hand out the number word cards to all the children in the class. Hold up a numeral for all to see. Ask the child having the number word card representing this numeral to stand and show his card to the rest of the class.

**Activity #2:** Reverse the procedure of the above activity. Hand out numeral cards to the children. You hold up a number word card.

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**Stage of Development:** Semiabstract

**Equipment:** Cards showing number words and numerals representing the same numbers

**Activity:** Hand out three identical numeral cards for each number word card. This way three children will hold numeral cards for each number word. When you hold up a number word card, it then becomes a contest between the three to see who stands up first with the correct answer.

This activity could also be altered by giving the children the number word cards and the teacher holding up a numeral card.

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**Stage of Development:** Semiabstract

**Equipment:** Overhead projector

**Activity:** Write a number word on the overhead projector. Require a child to identify the number word, verbally. Repeat with other number words.

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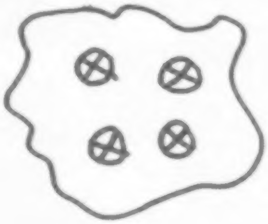
NOTE: Change the number words used in the activities to fit the level of study. Do not spend a great amount of time on each and every number word to one thousand, but spend your time teaching the basic units one, two, three, four, five, six, seven, eight, nine, ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred and thousand.

## FORMING SETS FOR NUMERALS

### Introduction

To aid in the child's understanding of the cardinal number of a set and the numeral associated with this number, the experience of forming sets having these numerals is of importance. Rational counting will also be strengthened with this activity.

The forming of sets enables the teacher to see if the child understands the meaning of a "4", which means a set of four elements.



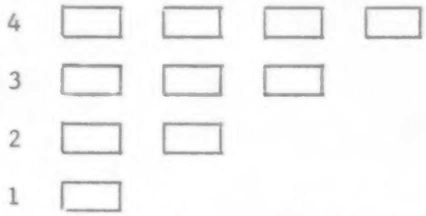
## CLASSROOM ACTIVITIES

**Concept:** Forming sets

**Stage of Development:** Concrete

**Equipment:** Large number line on floor, counting objects (blocks, etc.)

**Activity:** The number line is taped to the floor. Each child is given a set of objects. He must form a set of objects to the right of a numeral on the number line. The set must be equal to that numeral. This is continued until all the numbers have had sets made for them. This is an opportune time to discuss the concept of "one more than". This is an obvious concept when one compares the sets vertically.



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**Stage of Development:** Concrete

**Equipment:** Styrofoam cups with numerals written on their sides, counters (pegs, sticks, bottle caps, etc.)

**Activity:** Each child is given a set of counters and a cup. He is told to place the number of counters in the cup that is indicated by the numeral on its side. When he is finished, he must bring the cup to the front of the class and count out the counters to make sure they are correct.

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**Concept:** Forming equivalent sets

**Stage of Development:** Concrete

**Equipment:** Cards showing sets, counters (cubes, discs, etc.)

**Activity:** Each child receives a sufficient amount of counters. A card displaying a set is shown to the children. They are to form a set equivalent to the one shown on the card, using the counters given to them. The teacher checks to see if they are correct, then repeats the directions, using a different card.

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Concept: Forming sets - oral command

Stage of Development: Concrete

Equipment: Counters (cubes, sticks, buttons, etc.)

Activity: Each child is given several counters. A verbal description of the set that is to be formed, is then given. (E.g., "Make a set having six blocks.") After the children have formed their sets, the teacher should check to see if they are correct. New directions are then given and the process is repeated.

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Concept: Forming sets - written command

Stage of Development: Concrete

Equipment: Cards showing numerals, counters (cubes, sticks, discs, bottle caps, etc.)

Activity: Each child gets several counters. The teacher holds up a card showing a numeral. The children form a set for this numeral. It must have the number of elements described by the numeral.

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Concept: Forming sets

Stage of Development: Concrete

Equipment: Some method of making a set of sounds (drum, piano, bell, hammer and block of wood, metal triangle, two dowel rods, clapping hands, stamping feet, tapping the table, etc.), counters (cubes, discs, sticks, bottle caps, etc.)

Activity: Each child receives a set of counters. The teacher makes a set of sounds, using one of the above pieces of equipment. This is done after the children have closed their eyes. When they open their eyes, they are to form a set of counters whose cardinal number is the same as the set of sounds. After the teacher checks to make sure they are correct, he will make a new set of sounds and repeat the above procedure.

********************************

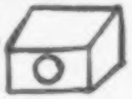
Stage of Development: Concrete

Equipment: Flash cards showing numerals or sets, some method by which the child can make a set of sounds (piano, bell, metal triangle, hammer and block of wood, etc.)

Activity: One of the cards is flashed to the student. If the card shows a set, the child will make a set of sounds equivalent to the set on the card. If it is a numeral, the child will also form a set of sounds having that number of elements.

*******************************

Stage of Development: Concrete

Equipment: A box which has a hole large enough for a child to place his hand inside the box , cards showing cubes, numerals, or sets

Activity: Several blocks are placed inside the box. The children are given instructions to reach inside the box and pull out a certain number of blocks. (E.g., "Please get me four blocks from the box.") The number must be kept rather small, because the children's hands are small.

*******************************

Stage of Development: Concrete

Equipment: Geo board, numeral cards, rubber band

Activity: Each child receives a geo board and a rubber band. He is either shown a numeral card or is given a numeral, verbally. The child then encloses a set of nails on his geo board, equal to the numeral given to him. The cardinal number of this set of nails should be the same as the numeral shown to the class.

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Stage of Development: Concrete

Equipment: Pegboard geo boards, pegs, numeral cards

Activity: Each child is given a pegboard geo board and a set of pegs. A numeral is stated, verbally, or a numeral card is shown to the class. The children will form a set of pegs on their pegboard geo board. The cardinal number of this set will be the same as the numeral given to them.

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Stage of Development: Concrete

Equipment: Pegboard number line, pegs, clothespins with numerals written on them

Activity: Only one child at a time may work on the pegboard number line. The non-numbered side is turned toward the child. The student is then given a box of pegs. A clothespin, whose numeral is exposed to the child, is placed on the number line. The child will then form a set of pegs equal to this numeral, using the pegboard number line.

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Concept: Forming sets - verbal command

Stage of Development: Semiconcrete

Equipment: Individual flannel boards, flannel objects

Activity: The teacher verbalizes the cardinal number of a set. The children all have a flannel board and objects. They must form, on their flannel boards, a set which has the cardinal number stated by the teacher.

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Concept: Forming sets - written command

Stage of Development: Semiconcrete

Equipment: Same as in the preceding activity, plus cards showing numerals

Activity: The children form sets on their flannel boards after seeing the number on the card held up by the teacher.

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Concept: Forming equivalent sets

Stage of Development: Semiconcrete

Equipment: Individual flannel boards, flannel objects, cards showing sets

Activity: The teacher holds up a card showing a set and the children form an equivalent set on their flannel boards.

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Concept: Forming sets - verbal command

Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic objects

**Activity:** The children are to form particular sets on the magnetic board as stated verbally by the teacher. (E.g., "Form a set of six characters on the magnetic board.")

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**Concept:** Forming sets - written command

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, objects, flash cards with numerals

**Activity:** A flash card is held up for all to see. Someone will then come forward and form a set on the magnetic board. This set must have for its cardinal number, whatever numeral was shown.

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**Concept:** Forming equivalent sets

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic objects, cards showing sets

**Activity:** A card is shown to the class. One child then forms a set on the magnetic board which would be equivalent to the one on display. The teacher might also draw a line dividing the board into halves, then place a set on one side while the child forms an equivalent set on the other side.

******************************

**Concept:** Forming sets - verbal

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards, cards showing sets or numerals

**Activity 1:** The children will form a set on their chalkboards (one which has been described to them verbally).

**Activity 2:** The children form a set on their chalkboards equivalent to the one shown them on a flash card.

(Note: When the children form sets, they should use the basic patterns for sets - the ones established earlier in this guide.)

******************************

Concept: Forming sets for numerals

Stage of Development: Semiabstract

Equipment: Overhead projector, pennies or flannel objects with which to form images, flannel numerals

Activity: A numeral is placed on the overhead projector and the students are asked to name it. Someone may then construct a set, representing the numeral, on the overhead projector. As the set develops, the number of each object must be said as it is placed on the overhead projector. When the correct amount is reached, the class should stop counting.

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Concept: Forming sets

Stage of Development: All stages

Equipment: Magnetic board, magnetic objects, flannel board, flannel objects, chalkboard, counting blocks, counting sticks

Activity: Five different centers can be set up in the classroom. These centers would be magnetic board, flannel board, chalkboard, counting blocks, and counting sticks. Two children are placed at each center. The children are then told to form a set which has a definite number of elements. Those children who are not at a center are to see if any of the children at the centers form an incorrect set. If they see an error they may go to the center and correct the mistake. The children at the centers are given several problems; then they change places with the children who are seated.

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## MATCHING NUMERAL WITH SET

### Introduction

As soon as pupils can readily recognize and name the number of a group, they can be shown the numeral (e.g., 3) that tells how many objects are in the group.

In the previous section, the student was required to form sets for given numerals. This section emphasizes this understanding in reverse (i.e. Matching numeral with set).

## CLASSROOM ACTIVITIES

Concept: Matching numeral and set

Stage of Development: Concrete

Equipment: Individual decks of cards showing numerals, 6" cubes whose sides show sets

Activity: The teacher rolls a cube, exposing the set on its top side. Each child has a deck of cards showing numerals. Each one is expected to choose the numeral representing the cardinal number of the set exposed and then to show it to the teacher.

This is repeated by throwing the cube several times.

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Stage of Development: Concrete

Equipment: Large tagboard cards showing sets, large tagboard cards showing numerals

Activity: The cards are arranged to show the numerals in a row (in random order). Cards showing the sets for these numerals are arranged in a row parallel to the one showing the numerals. (These, too, will be in random order.) A child is told to stand by a numeral and then to walk to the set showing the same number of elements. This is repeated with many children.

******************************

Stage of Development: Concrete

Equipment: Individual decks of cards showing numerals, counting objects

Activity: Each child is given a deck of numeral cards. A set of objects is placed on a table so all can see it. The children then select from their decks the card that shows the numeral which represents the set. The cards are held up for the teacher's inspection.

******************************

Concept: Matching numeral and set

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel shapes, cards showing numerals (a set for each child)

Activity: Several different sets are placed on the flannel board. The students hold up the proper numeral card as the teacher points to a set. The cards are then checked to see that all agree. When the class agrees on the proper card, it is put beside the set. This procedure is repeated for several sets.

******************************

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel shapes, and numerals

Activity: A set of flannel shapes is placed on the flannel board. One student determines the number of elements in the set. Someone else selects the numeral and places it on the flannel board next to the set that it represents. The sets are changed and the activity continued until all have had an opportunity to participate.

******************************

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel objects, numerals, pieces of yarn

Activity: Different sets are formed on one side of the flannel board and numerals are placed on the other side. The children match the numerals and sets by connecting them with pieces of yarn.

******************************

Stage of Development: Semiconcrete

Equipment: Flannel board, pictorial representations of sets (with velour paper on the back so they will adhere to the flannel board), flannel numerals

Activity: Several equivalent sets are placed with other sets on the flannel board. The numerals for the equivalent sets are also placed on the flannel board. The children then rearrange the equivalent sets so they will be next to their cardinal numbers.

Stage of Development: Semiconcrete

Equipment: Two chairs, string, clothespins with numerals on them, different pieces of clothing cut out of construction paper and placed together to show sets

Activity: The string is tied between the two chairs to form a clothesline. The child takes a clothespin with a numeral on it and finds the pieces of clothing showing the set for this numeral. He then pins this set of clothing to the string. This should be repeated with several children.

********************************

Concept: Matching numeral with set

Stage of Development: Semiabstract

Equipment: Overhead projector, flannel numerals, and objects to be used on the projector

Activity: A set is formed on the overhead projector. A child selects the correct numeral for this set and places it on the projector for all the class to see. This activity is repeated with many sets.

********************************

Stage of Development: Semiabstract

Equipment: Overhead projector, pen

Activity: Several sets, containing different numbers of elements, are drawn on the overhead projector. The cardinal numbers of the sets are written below or to the side. Children then come to the overhead projector and draw lines connecting the numerals with the sets that they represent.

********************************

Stage of Development: Semiabstract

Equipment: Playing cards with the numerals cut off, boxes labeled with numerals

Activity: Each child is given one of the playing cards. A child is then called upon to bring his card to the front of the room, show it to the class, tell how many elements are in the set, and then place the card in the correct box. Every child should be given a chance to participate. Then each one is handed a new card and the above procedure is repeated.

********************************

Stage of Development: Semiabstract

Equipment: Number house, cards showing sets

Activity: The rooms of the house are labeled with numerals. A card is given to each child. The teacher then explains to the children that the cards live in the rooms of the house. The cardinal numbers of the sets are the room numbers in the house.

Each child must show his card to the class and place it in the correct room of the house. This is continued until all children have placed their cards in the house.

*******************************

Stage of Development: Semiabstract

Equipment: Cards showing sets, cards showing numerals

Activity #1: Hand out set cards to the children. Hold up a numeral. Require all children having a set representing this numeral to stand. Repeat with other numerals.

Activity #2: Hand out the numeral cards to the children. Hold up a set card. Children having a numeral card matching the set card must stand.

*******************************

Stage of Development: Semiabstract

Equipment: Cards showing numerals, cards showing sets

Activity: Hand out set cards to half the class and numeral cards for these sets to the other half. Ask the children to walk around the room until they find a child whose card matches theirs. (i.e. set matching numeral)

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## WRITING OF NUMERALS

### Introduction

It is wise to defer the writing of numerals for some time - even until the latter part of grade one, in some cases. Writing numerals calls for a level of physical maturation and coordination that many pupils do not have in kindergarten or the early part of grade one. Also, students must be able to recognize the numerals and their meanings before learning how to write them. Much practice in recognizing numerals and the number of objects in a given set is most desirable.

A first grade child should be able to write the numerals 0-9, and also understand the concept of place value, before he attempts to write numerals greater than nine.

# CLASSROOM ACTIVITIES

**Concept:** Writing numerals

**Stage of Development:** Concrete

**Equipment:** Beaded numeral cards

**Activity:** The children trace around the beaded numerals with their fingers. This gives them a chance to get the feel of the numerals.

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**Equipment:** Large styrofoam numerals

**Activity:** Each child should be given an opportunity to handle the numerals. This will help him to get a visual image that will be necessary when he tries to write the numerals for the first time.

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**Concept:** Writing numerals

**Stage of Development:** Concrete

**Equipment:** Instant pudding, finger painting paper

**Activity:** Use instant pudding mixture as finger paint material. Each child practices his numerals in his pudding.

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**Concept:** Writing numerals

**Stage of Development:** Semiconcrete

**Equipment:** Cards with sandpaper numerals 0-9

**Activity:** The child traces the sandpaper numeral with his finger in order to determine its form.

An alternate activity would be to have the child close his eyes and trace a numeral, then guess the identity of the numeral.

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# CLASSROOM ACTIVITIES

Concept: Writing numerals

Stage of Development: Semiabstract

Equipment: Overhead projector, pen

Activity #1: The children are to write numerals on the overhead projector when the teacher gives them a verbal description of the numeral. (E.g., "Make a six.")

Activity #2: One child writes the numerals from one to ten on the overhead projector. This should be repeated with many members of the class.

******************************

Stage of Development: Semiabstract

Equipment: Individual chalkboards, numeral cards

Activity: Several sets, with six through ten members, are drawn on the chalkboard. The students identify the number of members in each set by holding up their numeral cards. The teacher then points to the set containing six members and says, "How many members are there in a set?" A large 6 is written on the board, and the children are told, "We write 6 like this." The students may trace a large 6 in the air; then they may draw a 6 on their chalkboards. The same procedure is followed to teach seven, eight, nine and ten. (Note: This development takes time. Do not move too fast. Be certain students can recognize sets of six before developing the meaning of seven, etc.)

******************************

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity 1: The students will write a numeral given verbally by the teacher. Several numerals should be given.

Activity 2: The students write the numerals from 1 to 10 on their chalkboards. As the child matures in his ability to write numerals, the number of numerals can be increased.

Activity 3: One child begins counting with some particular number. The teacher stops him, and the class is then asked to write the numeral on which he was stopped.

********************************

Stage of Development: Semiabstract

Equipment: Individual chalkboards, domino flash cards

Activity: A flash card with a domino pattern is shown to the class. Each student is to write on his chalkboard the numeral that represents this pattern. He then holds it up for the teacher's observation. Several different domino patterns should be used.

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Concept: Writing numerals

Stage of Development: Abstract

Equipment: A sheet of paper sectioned into one hundred squares (10" x 10")

Activity: Each child is given one of these sheets of paper. He is to begin with 0 and write the numerals to 100, using the squares on the paper. The numerals must be in the correct sequence. When the student is finished, he has his own hundreds board.

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NOTE: Use the same activities for writing numerals to 1000, merely change the level of difficulty relative to the students. Once a student has demonstrated he can write the numerals at various levels, give him credit for understanding. He does not have to write out all the numerals.

# ORDER OF NUMERALS

## Introduction

The order of the whole numbers used in counting emerges naturally from work with sets. Beginning with the empty set, one member at a time is added (or joined). The cardinal numbers of the respective sets formed are a natural sequence of the whole numbers, each number being one greater than that in the previous set. Therefore, it is important to work with ordering sets before talking about the ordering of numerals.

The set of whole numbers is said to be ordered. This means that every non-empty subset of the whole numbers has a smallest element. For example, in the subset /2, 3, 4/ there is one element, two, which is less than any other element in the set. This property of ordering enables one to find the whole number between, before, and after a given whole number. This also means that with any two whole numbers one can tell which is less and which is greater.

A good understanding of the concepts of order will be essential before the child can understand the concept of a number line.

**Concept:** Order - sets

**Stage of Development:** Concrete

**Equipment:** Paper plates, boxes, objects to set on the plates

**Activity:** The students will help the teacher form sets by placing a specified number of objects on the plates. First, they will help form a set having three members, then one with two members, then one with eight members, etc., until each of the ten plates has a set. A representative set for each number from one through ten should be formed. Ten boxes, each labeled with a numeral, should be placed on the teacher's desk in natural order. The numerals will face the students. The teacher then holds up a paper plate so that the objects on it can be seen by all the students. The children then tell into which box the objects must be emptied so that the number of objects matches the numeral on the box. This activity is continued until each set is in the appropriate box (in natural order).

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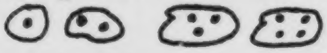
**Concept:** Order - sets

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic chalkboard, magnetic cards showing sets 0-10

**Activity:** The children arrange the set cards in natural order. (e.g.  etc.)

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**Concept:** Order of sets

**Stage of Development:** Semiabstract

**Equipment:** Several 3" x 5" cards for each student

**Activity:** The students draw or paste pictures of sets on the cards, so that each number, one through ten, is represented by a card. Then, they arrange the cards in natural order.

******************************

Concept: Order of numerals

Stage of Development: Concrete

Equipment: Children

Activity: Several children are arranged in a row. The teacher asks questions about the children who are in a certain position in the row. These questions should be concerned with the concepts of "between", "in front of", "coming before", "coming after", etc.

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Stage of Development: Concrete

Equipment: Paper cups that are numbered (1 - 10)

Activity: The cups are given to one student with these instructions: "Arrange the cups so the numerals will be in their correct order." This can be repeated with several children.

+ + + + + + + + + + + + + + + +

Stage of Development: Semiconcrete

Equipment: Paper cups that are numbered (1-10)

Activity: Cups one and two are arranged horizontally from left to right (the arrangement may begin next to an object, such as a block). The teacher then asks the children, "Which cup follows cup one?" They are told that, "Cup one is called the first cup. It is the first cup from the left (or the first cup from the block). Cup two is next to cup one. It is the second cup from the left going in the direction we read (left to right). We can also say that cup one is in the first position and cup two is in the second position." Cup three is added. The teacher says, "Which cup follows cup two? Cup three is called the third cup. Cup three can also be called the last cup of this arrangement because it has the highest number. It is in the third position in this arrangement." The other cups (4-10) may be added in the same way.

+ + + + + + + + + + + + + + +

**Concept:** Order - numerals

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, cards showing numerals (with small magnets glued to the backs)

**Activity:** The cards are given to a child with the instructions that he must arrange them on the magnetic board in their proper order. If he does it incorrectly, someone else may help him do it correctly. When the child has finished, the cards are taken down from the magnetic board and another child is given an opportunity to arrange them in order. A discussion of the phrases "between", "one more than", "before", "after", and "one less than", can be used as the numerals are placed on the magnetic board.

+ + + + + + + + + + + + + + +

**Concept:** Order - numerals

**Stage of Development:** Semiabstract

**Equipment:** Cards showing numerals

**Activity:** The teacher asks seven students to hold up the numeral cards, zero through six, in random order. One student is then told to turn around (e.g., the student holding the two card). The teacher asks what numeral is missing. This activity should be repeated with other numerals. Perhaps two students can be asked to turn around at the same time.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Cards showing numerals

Activity: Several students are asked to hold the numerals in proper sequence. Two or more of the students are then asked to change places with one another. The class then rearranges the students so that the cards are in natural order. This can be repeated, increasing the number of numerals out of place.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Pocket chart, cards showing numerals

Activity #1: The cards are mixed up and then given to a child to place in the correct order in the pocket chart. If done correctly, the cards are taken out and another child is given a chance to arrange them correctly. This activity should be repeated several times.

Activity #2: The cards are placed in their correct order in the pocket chart. The children then close their eyes as the teacher hides one of the cards. They must determine which card has been removed. This is repeated as cards are removed from several different positions on the board. The teacher should also have the children point out numerals which have been described by the position terms "before", "after", "between", "one more than", "one less than", etc.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Cards showing numerals (a set for each child)

Activity: The children are asked to arrange the numerals in their proper order (using the floor or desk top). All children should be working on this at the same time.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Two chairs, string, clothespins, various pieces of clothing cut out of construction paper (each one with a numeral written on it)

Activity: The student must put the clothes on the line so the numerals will be in the correct order, counting from left to right. This should be repeated with several children.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: Children are required to write in proper sequence a set of numerals. These numerals may be one, two, three or four digit numbers.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Show-me-boards

Activity: Teacher writes a specific numeral on the chalkboard. The children are required to form a certain numeral on their show-me-boards as directed by the teacher. (e.g., Teacher writes 360 on chalkboard. Child must show the numeral that is "one less than", or "one more than", etc.)

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Hundreds board, catchers mitt

Activity: Children are asked to catch certain numerals on the hundreds board as directed by the teacher. (e.g., Teacher asks child to catch the numeral that is two less than 34.)

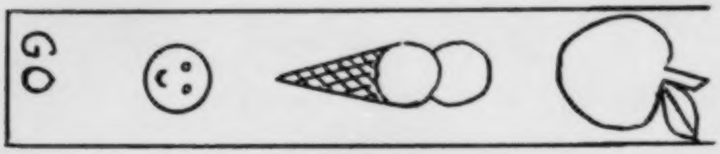
+ + + + + + + + + + + + + + +

NOTE: Use the same activities for order of numerals to 1000, merely change the level of difficulty to match your students' abilities. Spot check your students for understanding. Do not require them to arrange 1000 numerals into the proper sequence.

Concept: Number line

Stage of Development: Concrete

Equipment: 4 yard pictoral number line (pictures cut from magazines to represent numerals; for zero use the word "start" or "go")



extend to 10 objects

Activity #1: The teacher initiates a discussion about walking along a path. The children are asked to stand on certain pictures.

Activity #2: The children are asked to stand on a certain picture and then to move forward or backward from this picture. Several children should have a chance to participate.

Activity #3: Teacher now introduces terminology such as starting at go (the beginning of the number line), only moving one space at a time and following simple 1, 2, 3 step directions such as: "Stand on the apple, go back one step and tell us what you're standing on."

+ + + + + + + + + + + + + + + +

Concept: Number line

Stage of Development: Semiconcrete

Equipment: Magnetic board and magnetic magazine cutouts

Activity: The teacher uses identical activities mentioned above. This time the children make their moves using their fingers and not their bodies.

+ + + + + + + + + + + + + + + +

**Concept:** Number line

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, transparency showing a number line with objects, pop bottle cap as a mover

**Activity #1:** The teacher gives verbal directions and a child (using the pop bottle cap) demonstrates the direction given.

**Activity #2:** A child demonstrates his own number line story and the children give the verbal explanation.

+ + + + + + + + + + + + + + +

**Concept:** Number line

**Stage of Development:** Abstract

**Equipment** Worksheets and pictoral number line

**Activity:** Explanation: There are six worksheets designed to be used in sequential order. Some students may not need all the worksheets.

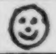
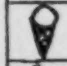
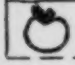
Worksheet #1

NOTE: Child draws correct picture in empty space.

| Go | # of Steps | Stop |
|---|---|---|
| Go | → → | |
| Go | → | |
| Go | → → → | |

etc.

Worksheet #2

| Go | # of Steps | Stop |
|---|---|---|
|  | → | |
|  | → → | |
|  | → → → | |

etc.

Worksheet #3

| Go | # of Steps | Stop |
|---|---|---|
| 🍎 | ← |  |
| 🍦 | ← ← |  |
| ☺ | ← |  |

Worksheet #4

NOTE: Child fills in blank with arrows.

| Go | # of Steps | Stop |
|---|---|---|
| ☺ |  | 🍎 |
| 🍦 |  | ☺ |
| 🍎 |  | ☺ |

Worksheet #5

| Go | # of Steps | Stop |
|---|---|---|
|  | → → | 🍎 |
|  | → | 🍦 |
|  | → | ☺ |

Worksheet #6

NOTE: Culminating worksheet.

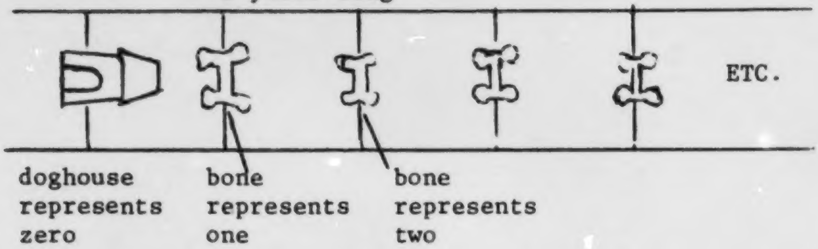
| Go | # of Steps | Stop |
|---|---|---|
| Go | → → → |  |
| ☺ | → → |  |
| 🍎 | ← |  |
| 🍦 |  | 🍎 |
|  | → → | 🍦 |

+ + + + + + + + + + + + + + +

Concept: Number line

Stage of Development: Concrete

Equipment: Doghouse and 10 dog bones; segmented number line 5 yards long



Activity #1: Child pretends to be Snoopy and stands on the doghouse. The teacher gives the verbal instructions to pick up a certain number of bones. After picking up the bones, the child counts them to see how many he has in his hand. (This number will later represent the numeral the child will stand on when given the verbal direction.)

Activity #2: Repeat the above activity but when the child counts his bones have him pick up so many more bones. He then has to tell how many he has altogether. He then tells the number story. (By doing this, he is joining sets.)

Activity #3: Starting at the doghouse, the child picks up so many bones. He then holds them in his hands and stands on the last empty segmented line. He then pretends to bury the bones by putting them back. He tells how many he had to begin with, how many he buried, and how many he has left. (By doing this, the child is separating sets.)

NOTE: This activity can be extended through all four stages of development.

+ + + + + + + + + + + + + + +

**Concept:** Number line

**Stage of Development:** Concrete

**Equipment:** Segmented line

**Activity:**

1. Child moves forward on line.
2. Child moves backward on line.
3. Child chooses another friend to take his place.
4. This child moves forward; the teacher records on the chalkboard what has happened:

   $\text{Jimmy} \xrightarrow[\textcircled{m}]{}$ or $\text{J} \xrightarrow[\textcircled{m}]{}$

5. The same child moves backward; the teacher records on the chalkboard what has happened:

   $\text{Jimmy} \xleftarrow[\textcircled{m}]{}$ or $\text{J} \xleftarrow[\textcircled{m}]{}$

6. Let several children do this.

+ + + + + + + + + + + + + + + +

**Concept:** Number line

**Stage of Development:** Semiconcrete

**Equipment:** "Bee-in-the-flower-garden" number line

**Activity:**

1. Child moves bee forward from flower to flower.
2. Child moves bee backward from flower to flower.
3. Child chooses a friend to take his place.
4. This child moves forward and the teacher records what has happened.

   $\text{blue zinnia} \xrightarrow[\textcircled{m}]{} 2 = \text{orange zinnia}$

   or: $\text{BZ} \xrightarrow[\textcircled{m}]{} 3 = \boxed{\text{OZ}}$

5. This child moves backward and the teacher records what has happened.

   $\text{orange zinnia} \xleftarrow[\textcircled{m}]{} 6 = \text{yellow zinnia}$

   or: $\text{OZ} \xleftarrow[\textcircled{m}]{} 6 = \boxed{\text{YZ}}$

+ + + + + + + + + + + + + + + +

Concept: Number line

Stage of Development: Semiabstract

Equipment: Transparency of number lines

Activity: Cover the three bottom lines. Have a child move forward on the line and backward. The teacher records:

$$J \xrightarrow[\textcircled{m}]{} \quad \text{or} \quad J \xleftarrow[\textcircled{m}]{}$$

Show the flower garden. Have a child move forward and the teacher records what happened:

$$\text{green zinnia} \xrightarrow[\textcircled{m}]{} 3 = \boxed{YT}$$

Have the child move backward. Teacher records what has happened:

$$\text{yellow tulip} \xleftarrow[\textcircled{m}]{} 5 = \boxed{YZ}$$

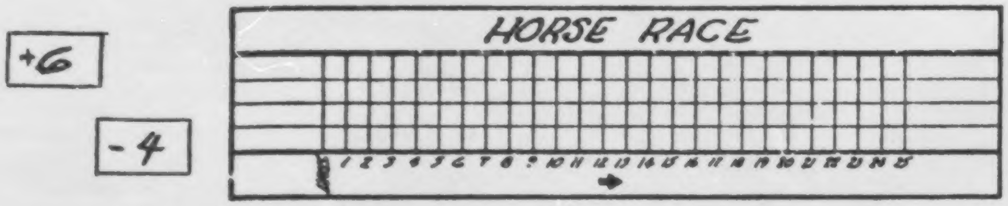
Show the third number line. Have child move forward. Have child move backward. Teacher records results on the chalkboard. Don't show the bottom number line at this point.

+ + + + + + + + + + + + + + + +

Concept: Number line (0-25)

Stage of Development: Semiconcrete

Equipment: Make a race track on a strip of oaktag or railroad board. Mark it into four lanes and number it as illustrated in the diagram. Mark a set of 27 3" x 5" cards so that 18 show +1, +2, +3,...+9, with two cards for each; and nine show -1, -2, -3,.... -9. You will also need a marker for each player.



**Activity:** Horse Race

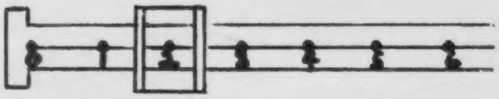
1. Each player puts his marker at the starting line. One player shuffles the addition and subtraction cards and puts them face down.
2. The first player takes the top card and reads what it says. If it is an addition card, he moves his marker along the track the distance it indicates. If it is a subtraction card, he waits for his next turn. (A player cannot begin until he gets an addition card.)
3. Play continues with players taking turns in order. Each time a player draws an addition card, he moves it to the right the distance indicated by the sum. He moves it to the left the distance indicated by the difference when he draws a subtraction card.
4. The winner is the player who reaches 25 first.

+ + + + + + + + + + + + + + + +

**Concept:** Number line

**Stage of Development:** Semiabstract

**Equipment:** Number line slide rule for each child



2 two-foot number lines taped to the chalkboard.

**Activity:** The class is divided into two groups. One child from each group takes a turn going to the board. The teacher tells a number story. The two students at the board race to be the winner while the rest of the students use the slide rules.

+ + + + + + + + + + + + + + + +

Concept: Number line

Stage of Development: Abstract

Equipment: Worksheet 1, 2, 3, 4, 5, 6, 7, 8, and 9 (NOTE: These are found in the overhead kit or at the math office, 682-4598.)

Activity: Show worksheet. Explain. Have children color flowers first, then do equations.

+ + + + + + + + + + + + + + +

## CLASSROOM GAME

**Concept:** Order

**Equipment:** Hundreds board

**Activity:** The hundreds board is displayed so all can see it. The children are asked to solve mathematical statements such as the following:

$$21 \longrightarrow \longrightarrow \longrightarrow \uparrow \longleftarrow = \boxed{\phantom{00}}$$

The symbols mean:

$\longrightarrow$ One numeral to the right

$\longleftarrow$ One numeral to the left

$\uparrow$ Up one numeral

$\downarrow$ Down one numeral

Therefore, the answer to the above problem is 13. Many different math problems, similar to the one shown above, should be presented.

An alternate activity would be to give the child two numerals and require him to supply the symbols needed to get from one numeral to the other. (E.g., 20__________=32 could be $20 \longrightarrow \longrightarrow \downarrow =32$.)

# NUMBER COMPARISONS

## Introduction

The set of whole numbers is said to be ordered. This means that every non-empty subset of whole numbers has a smallest element, a largest element and the others can be arranged in a sequence relative to their value. This property of ordering enables one to find the whole number between, before and after a given whole number. This also means we can compare any two different whole numbers to determine which one is less and which is greater than the other.

## EVEN OR ODD NUMBERS

When children count the objects in a set, they can arrange the objects in groups of twos, so that it becomes obvious that the groupings result in either one object left over or no objects left over. If there is one object left over, there is an odd number of objects in the set. If none are left over, the set contains an even number of objects.

## PRIME NUMBERS

There is one set of whole numbers that are only divisible by themselves and one. These numbers are called prime numbers.

Concept: Number comparisons

Stage of Development: Concrete

Equipment: Large number line on floor

Activity: The number line is taped to the floor. Then, the children are asked to stand on certain numerals. (e.g., "Stand on the numeral six.") As the teacher describes numerals on which the children are to stand, he uses these terms: "one more than"; "one less than"; or "between".

1. "Stand on the numeral that is one less than five."
2. "Stand on the numeral that is between five and seven."
3. "Stand on the numeral that is one more than three."

+ + + + + + + + + + + + + + +

Equipment: Large, stand up styrofoam numerals

Activity #1: The order of the numerals is mixed. A student comes forward and rearranges the numerals so they are in the correct order. The children then turn their backs and the teacher again mixes the numerals. Another student fixes the order so it is again correct. This should be repeated several times.

Activity #2: The numerals are arranged in correct order. The teacher questions the children about numerals that are in a certain position.

1. "Show me the numeral that comes before the numeral six."
2. "Show me the numeral that is between four and six."
3. "Show me the numeral that comes after three."
4. "Show me the numeral that is one more than the numeral eight."
5. "Show me the numeral that is one less than five."

Stage of Development: Concrete

Equipment: Plastic bottles with numerals painted on their sides

Activity: The activity used for the stand up styrofoam numerals is repeated.

+ + + + + + + + + + + + + + +

## CLASSROOM ACTIVITIES

**Concept:** Greater than, less than

**Stage of Development:** Concrete

**Equipment:** Baby food jars of rice, beans, corn, etc., shoe box, cards showing the words "more", "less", "less than", and "greater than"

**Activity 1:** Pairs of jars are filled with the same commodity (rice, beans, corn, etc.). They must be filled so that they have obviously different amounts. A side is cut off of the shoe box and a divider placed in the middle of the box. The jars having the same commodity are placed in the two compartments of the shoe box for all to see. A child verbalizes or places the cards showing "more" and "less" on the compartments having the most and the least amounts of the commodity. This should be repeated with jars containing different commodities.

**Activity 2:** A student places a card showing "less than" or one showing "greater than" on the divider between the commodities-making a true statement.

******************************

**Stage of Development:** Concrete

**Equipment:** Two paper plates, counters (blocks, sticks, discs, etc., cards showing "more", "less", "greater than", "less than"

**Activity 1:** A different number of counters is placed in each plate. A student puts the cards showing "more" and "less" on the plates having the most and the least number of counters. This is repeated several times, using different counters.

**Activity 2:** Counters are put on the two plates. A child places the card showing "greater than" or the one showing "less than" between the plates, to make a true statement.

******************************

Concept: Greater than - less than

Stage of Development: Semiconcrete

Equipment: Magnetic board or flannel board, objects, flannel and magnetic pieces showing "greater than" and "less than"

Activity 1: Two sets of objects are placed on the board. The phrase "greater than" or "less than" is put between the sets to make a true statement on the child may verbalize the statement.

Activity 2: One set and one word card are placed on the board
(e.g., ★★ less than.) A child forms a set to
★★
make this a true statement.

******************************

Concept: Greater than - less than

Stage of Development: Semiabstract

Equipment: Overhead projector, small transparencies showing the words "greater than", "less than", and objects for the overhead projector

Activity : Two sets of objects are formed on the overhead projector. They must have obviously different numbers of elements. Someone places the transparency showing "greater than", or the one showing "less than", between the sets to make a true statement.

******************************

| | |
|---|---|
| Concept: | Greater than, less than |
| Stage of Development: | Semiconcrete |
| Equipment: | Individual flannel boards, objects, flannel pieces showing words greater than, less than |
| Activity 1: | A set of objects is placed on the demonstration flannel board. The children then are told to form a set on their flannel boards that has "more" objects than the one on the demonstration board. After checking their results, the teacher asks them to form a set that has "fewer" characters. This can be repeated with different sets being placed on the demonstartion flannel board. The phrases "greater than" and "less than" can also be used. |
| Activity 2: | Two sets of objects are placed on the demonstration flannel board--leaving a space between the sets The children put the word card greater than or less than on their flannel boards, making a true comparison of the cardinal numbers of the two sets. Their boards are then held in the air for the teacher's observation. |

******************************

## CLASSROOM ACTIVITIES

**Concept:** Even and odd numbers

**Stage of Development:** Concrete

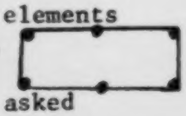
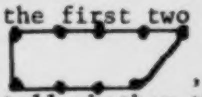
**Equipment:** Counting objects (cubes, sticks, discs, etc.)

**Activity:** Each child is given a set of counting objects. The number of objects may vary for different children. The children must determine whether they have an even number of objects or an odd number of objects. They are told that if they group the objects in pairs and there are no objects "left over", they have an even number of objects. If they end up with one object "left over", then they have an odd number of objects. Each student is asked to tell if he has an even or an odd number of objects and to state the number that represents the set.

*******************************

**Stage of Development:** Concrete

**Equipment:** Geo boards and rubber bands, or pegboards and pegs

**Activity:** A rubber band is placed around some of the elements on the first two rows of a geo board (e.g.,  or , etc.). The children are asked to tell whether the number of nails enclosed is even or odd. They must do this without counting the nails. This will be easy if the child realizes that an even number of nails yields a rectangle. The rubber band will be stretched into the shape of a rectangle because of the groupings by twos. After a child has determined whether the number of nails enclosed is even or odd, he must give the cardinal number of the set of nails. This should be repeated using several examples. If the pegboard is used, pegs are placed in the first two rows.

*******************************

**Concept:** Even and odd numbers

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, flannel board, objects

**Activity:** Several objects are placed either on the magnetic board or the flannel board. The students determine whether there is an even or an odd number of objects. This can be determined by grouping the objects in twos, as described in the introduction to this section. The children should also state the cardinal number of the set.

If individual flannel boards are used, each child should be given several objects.

*******************************

**Concept:** Even and odd numbers

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, objects

**Activity:** Several objects are placed on the overhead projector. Then the procedure described in the preceding activity can be used.

*******************************

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity 1:** The children are told to draw an even or an odd number of figures ( △ , ✱ , ● , etc.) on their chalkboards for the teacher's observation.

**Activity 2:** The children are asked to write the numeral for an even or an odd number on their chalkboards. The answers are held up for all to see.

*******************************

**Stage of Development:** Semiabstract

**Equipment:** Large number line on floor

**Activity:** The teacher should explain, and demonstrate, to the children that they can locate all the even numbers on the number line by beginning with the first one (zero) and then stepping over the next number they come to as they walk forward. This way they step on two, four, six, eight, etc. They can find the odd numbers by beginning with one and walking forward in a similar manner.

Several children can then demonstrate this by walking on the number line. They are asked to stop at different times and tell the number on which they are standing.

*******************************

**Stage of Development:** Semiabstract

**Equipment:** Flash cards showing numerals

**Activity:** A numeral is flashed to the class. The teacher calls on someone to tell whether the numeral is even or odd. For a small group contest, the children are seated in a semicircle and the cards are flashed to them. The first one to correctly identify the "evenness" or "oddness" of the number, gets the card. The child with the most cards at the end of the contest is declared the winner.

*******************************

**Stage of Development:** Semiabstract

**Equipment:** Show-me-boards, chalkboards

**Activity:** Teacher asks the child to form or write a specific numeral on their show-me-board or chalkboard. The child then must determine if it is an even or an odd number. (Key - observe the numeral in the ones place.)

*******************************

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, chalkboard

**Activity:** Write a random selection of two or three digit numerals on the overhead projector or chalkboard. The children are asked to circle all the numbers that are even numbers.

*******************************

Concept: Number comparisons - prime numbers

Stage of development: Semiconcrete

Equipment: Magnetic board, magnetic numeral cards

Activity: Place several one and two digit numeral cards on the magnetic board. Make sure some of the numbers are prime. A student should be asked to find a prime number on the board. Other students should be asked to identify prime numbers on the board until they have all been identified. Repeat the activity with different numerals and students.

+ + + + + + + + + + + + + + +

Concept: Number comparisons - prime numbers

Stage of development: Semiabstract

Equipment: Chalkboard or overhead projector

Activity: Same as above except write the numerals on the chalkboard or overhead projector. The children then are asked to identify the numerals that represent prime numbers.

+ + + + + + + + + + + + + + +

Concept: Number patterns

Stage of development: Semiconcrete

Equipment: Magnetic board, magnetic numeral cards

Activity: Arrange four of the numeral cards in a partial sequence on the magnetic board.

$\boxed{3}, \boxed{6}, \boxed{9}, \boxed{12}$

Require a student to select the numeral card that would be the next numeral in the sequence, namely $\boxed{15}$. Repeat the activity using other partial sequences such as:

$\boxed{4}, \boxed{8}, \boxed{12}, \boxed{16}$, ____ or

$\boxed{16}, \boxed{14}, \boxed{12}, \boxed{10}$, ____ or

$\boxed{6}, \boxed{3}, \boxed{6}, \boxed{3}$, ____ or

$\boxed{1}, \boxed{3}, \boxed{5}, \boxed{7}$, ____ Vary the activity by leaving out one of the terms in the pattern.

e.g. $\boxed{6}\,,\ \boxed{12}\,,\ \boxed{\phantom{00}}\,,\ \boxed{24}\,,\ \boxed{30}$ or

$\boxed{13}\,,\ \boxed{10}\,,\ \boxed{7}\,,\ \boxed{\phantom{00}}\,,\ \boxed{1}$

+ + + + + + + + + + + + + + + +

Concept: Number patterns

Stage of development: Semiabstract

Equipment: Overhead projector or chalkboard

Activity: Same as above except write the partial number pattern on the overhead or chalkboard and ask the students to write the missing term in the missing spaces.

+ + + + + + + + + + + + + + + +

Concept: Number patterns

Stage of development: Semiabstract

Equipment: Individual chalkboards

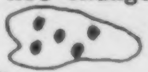
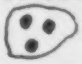
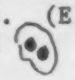
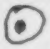
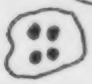
Activity: Ask the students to devise a number sequence and write it out on their chalkboards. The students then hold their chalkboards in the air for teacher observation and approval. Continue with other sequences.

+ + + + + + + + + + + + + + + +

# CONSERVATION OF SETS

## Introduction

### Number - Numeral

The principle of conservation of sets is that the cardinal number of a set does not change when that set is separated into two subsets. (E. g., The set  may be separated into the subsets  and  or  and  or their reversals)

The conservation of sets section of the skill sheet is subdivided into the categories (1-6), (1-12), and (1-18). You may want to gear your instruction to these various subdivisions.

## CLASSROOM ACTIVITIES

Concept: Conservation of sets

Stage of Development: Concrete

Equipment: Two hula hoops

Activity: Several children are chosen to be members of a set (3, 4, 5, 6, 7, 8, 9, or 10). The hula hoops are placed in such a way that they do not overlap. One child can separate the chosen set of children into two sets, by having them stand inside the two hula hoops. The teacher asks the class if there is a different way in which the chosen set could have been separated. The set should then be separated in different ways. This activity can also be carried out with two, large, non-overlapping circles drawn on the floor.

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Stage of Development: Concrete

Equipment: Counters (sticks, blocks, discs, buttons, etc.)

Activity: Each student receives a certain number of counters (3, 4, 5, 6, 7, etc.). He is told to arrange them in two sets, stating the total number of counters. All the students will then show how they have arranged the elements in the two sets. The activity should continue until all possible combinations of this set have been found. After each combination is found, the child must state the total number of the counters in the two sets. This number will remain constant.

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Stage of Development: Concrete

Equipment: Demonstration pegboard, pegs, rubber bands

Activity: A set of pegs (3, 4, 5, 6, etc.) is placed on the board, in random order. Two rubber bands can be used to divide this set into two, separate, non-overlapping sets. One child then arranges the rubber bands to form two different sets. This can be repeated until all set combinations have been exhausted. After each separation, the child should be required to state the total number of pegs on the board (illustrating the fact that this number does not change because of grouping).

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**Stage of Development:** Concrete

**Equipment:** Individual geo boards, three rubber bands

**Activity:** The children are to enclose a certain number of nails with a rubber band (e.g., six). The other two rubber bands are to be used to make two sets out of the set of enclosed nails (e.g., two and four). Each child then gives the set sentence for his geo board. (E.g., "Two nails and four nails makes six nails.")

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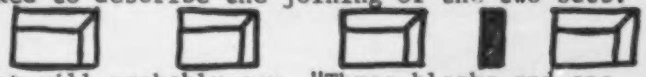
**Stage of Development:** Concrete

**Equipment:** For each student: four blocks, a marker for separating the blocks

**Activity:** Each student is given four blocks and a marker. The students use their markers to make as many different pairs of sets as possible with their four blocks. (One pair might include the empty set.) They are then asked to describe the joining of the two sets. (E.g.,  ). A student will probably say, "Three blocks and one block makes four blocks." This should be continued until all combinations are found.

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**Concept:** Conservation of sets

**Stage of Development:** Semiconcrete

**Equipment:** Individual flannel board, flannel shapes

**Activity:** Each child is given a set of shapes (3,4, 5, 6, etc.) to use on his flannel board. The children are instructed to form two groups on their flannel boards, using the flannel shapes given to them. The teacher then walks around the room to observe the different groupings. He can hold up flannel boards with different groupings, or unusual patterns, for class recognition. It should be emphasized that the total number of objects remains the same, regardless of the groupings. This activity can be repeated using different sets. The children verbalize the set sentence associated with each of their solutions. When the child is older, and more mature, he can show the number sentence associated with the joining of the two sets, using his "show-me-board" or an individual chalkboard.

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**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic objects

**Activity:** A set of objects is placed on the magnetic board (3, 4, 5, 6, 7, etc.). It is separated into two sets. One child may then give a verbal description of the set sentence associated with the two sets. (If the sets are (★★) and (★★★), the child should say, "Two stars and three stars makes five stars.") Other groupings might be (★) and (★★★★), ( ) and (★★★★★), (★★★) and (★★), or (★★★★) and (★). Each time, the child should be required to give a verbal description of the set sentence for the new grouping. At a later stage of development, the child can be required to give a number sentence for the grouping.

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**Concept:** Conservation of sets

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, pennies, or overhead projector objects

**Activity:** A set of objects is placed on the overhead projector. The class must determine the number of objects. A sheet of construction paper can be used to shield the objects from view, while the teacher arranges them into two distinct sets. When the construction paper is removed, the sets are exposed to the class. Someone is then asked to verbalize the set sentence for the joining of these sets. The coins may also be covered with the paper and then rearranged to form two different sets. The set sentence of these sets should be stated. Stating the set sentence for each new separation brings out the concept of conservation, for the total remains the same regardless of the groupings.

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**Stage of Development:** Semiabstract

**Equipment:** Chalkboard, individual chalkboards

**Activity:** A set is formed on the chalkboard (3, 4, 5, 6, or more elements). The children are then asked to make two sets on their individual chalkboards, with the two sets having a total number of elements equal to the set on the chalkboard. Recognition should be given to those having different combinations. This can be done by holding their chalkboards in the air for all to see. The children are asked to verbalize the set sentence for each combination. A different set can be put on the board and the above procedure repeated.

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Concept: Conservation of sets

Stage of Development: Abstract

Equipment: Paper, pencil, chalkboard

Activity: The activity is the same as the preceding one, except that the sets are formed on paper rather than on the individual chalkboards.

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Concept: Conservation of sets

Stage of Development: Semiconcrete

Equipment: Pocket chart, domino patterns

Activity: The domino patterns are placed in the pocket chart for all to see. Two children stand behind a line which is 10-15 feet from the pocket chart. The teacher then states a number that represents the number of dots on a domino. The first child that walks to the pocket chart and shows the domino for the number stated, is the winner.
(Note: For this particular competition, the teacher may want to have boys against girls or have the winner stay up until defeated.)

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Concept: Conservation of sets

Stage of Development: Semiconcrete

Equipment: Conservation board, pencil, paper, beads

Activity: The child is given the conservation board with the instructions that he must find all the possible combinations for the given set of beads. He writes down each new combination as he finds it. (If he is given seven beads, he should find these combinations: 0 and 7, 1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, 6 and 1, 7 and 0.)

# JOINING SETS

## Introduction

Joining sets is a natural activity for kindergarten children. The learning of addition begins by having the students handle objects. They first identify the number of objects in the sets that are to be joined, and then the number of the set that is formed. Essentially, the child learns to verbalize addition statements, such as, "A set of two blocks joined with a set of three blocks makes a set of five blocks."

In the above statement, the child is learning to verbalize "addition facts" by learning to "associate" cardinal numbers with their sets. At the kindergarten level, the child verbalizes addition. When he enters first grade, he learns to read and write what he has been saying.

Set and number operations, though relative, are not the same. When students work with objects they should think in terms of joining and separating sets, using appropriate words, such as, "join", "take away", "remove", and "leaves". When working with numbers, students should think in terms of addition and subtraction, using such words as add, subtract, plus, minus, and equals. For example:

Set sentence - A set of two dogs joined with a set of three dogs makes a set of five dogs.

Number sentence - Two plus three equals five

As a culminating activity, teachers may write experience charts with children dictating story problems. Story problems may be duplicated for all children.

## CLASSROOM ACTIVITIES

**Concept:** Joining sets - verbal response

**Stage of Development:** Concrete

**Equipment:** Two circles drawn on the floor

**Activity:** Boys are chosen for one circle and girls for the other circle. One of the students then states the number of boys in the one set and the number of girls in the other. He also states the number of children in the two combined sets.

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**Stage of Development:** Concrete

**Equipment:** Two hula hoops

**Activity:** The hula hoops are placed in the front of the room. Boys are put in one hoop and girls in the other. One child describes the number of boys, girls, and children in the sets.

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**Stage of Development:** Concrete

**Equipment:** Two trays, counters (blocks, discs, cubes, buttons, bottle caps, etc.)

**Activity:** A set of counters is put in each tray. One pupil is chosen to state the number of counters in each set and the total of the two combined sets. This activity can be repeated with different sets and counters.

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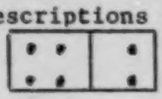
**Stage of Development:** Concrete

**Equipment:** Cards showing domino patterns, counting discs

**Activity:** The teacher exposes a domino card to the class. Each member of the class makes a copy on his desk, using the counting discs. The children then respond by giving verbal descriptions of the joining of these sets. (E.g.,  - the response would be, "Four and two makes six.")

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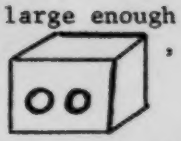
**Stage of Development:** Concrete

**Equipment:** A box having two holes large enough for a child's hands to be inserted , counting blocks, cards showing numerals or sets

**Activity:** Several blocks are placed inside the box. One child can then put his hands inside the box and pull out a certain number of blocks, holding some in each hand. The number that he is to pull out may be stated verbally or it can be shown with a flash card.

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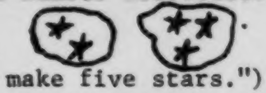
**Concept:** Joining sets - verbal response

**Stage of Development:** Semiconcrete

**Equipment:** Flannel board(s) and objects

**Activity:** Two sets of objects are formed on the flannel board(s). The children are to verbalize the cardinal number of each set and the number associated with the combined sets. (E.g., . The child should say, "Two stars and three stars make five stars.")

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**Stage of Development:** Semiconcrete

**Equipment:** Individual flannel boards, large flannel board, flannel objects

**Activity:** The teacher forms two sets on the large flannel board, while the children copy the patterns on their flannel boards. They are then required to verbalize the cardinal number of each set and the combined sets.

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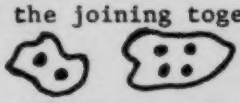
**Stage of Development:** Semiconcrete

**Equipment:** Magnetic chalkboard, shapes, chalk

**Activity:** Two enclosed curves are drawn on the board. Different shapes are placed inside these curves, forming two sets. A child may state the cardinal number of the set formed by the joining together of these two sets. (E.g., , "Two and four makes six.")

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**Stage of Development:** Semiconcrete

**Equipment:** Magnetic chalkboard, pictures of sets, small magnets

**Activity:** The small magnets are used to hold the pictures of two sets on the magnetic board. The children must identify the cardinal number of each set and of the combined sets. They should use language such as the following: "A set of two rabbits and a set of six rabbits makes a set of eight rabbits."

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Concept: Joining sets - verbal response

Stage of Development: Semiabstract

Equipment: Overhead projector, pennies, objects to show images on the screen

Activity: Two sets are formed on the overhead projector. The children are to verbalize the cardinal number of each set and of the combined sets.

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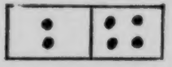
Stage of Development: Semiabstract

Equipment: Overhead projector, transparencies showing domino patterns (  ,etc.), sheet of construction paper

Activity: A transparency is placed on the overhead projector. Half of the transparency should be covered with construction paper; half should be exposed. The children are to state the cardinal number of the set that is shown. The paper is then removed so the other half shows. The cardinal number of this set should be stated, then the two can be joined and the cardinal number of the combined sets can be given. This should be repeated with several transparencies, showing many different sets.

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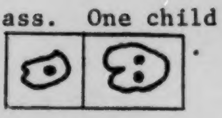
Stage of Development: Semiabstract

Equipment: Cards showing two sets

Activity: A card is shown to the class. One child verbalizes the set solution. (E.g.,  . The child says, "A set of one dot and a set of two dots makes a set of three dots.")

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Stage of Development: Semiabstract

Equipment: Set dominos

Activity: Each end of the domino should be used as a flash card for set recognition. Then the entire domino can be exposed for the joining of the sets. The cardinal numbers of each set should be found and verbalized.

An alternate activity would be to expose the entire domino to the class and require only the cardinal number of the combined sets to be given. A contest could result to see who could be the first to identify the cardinal number.

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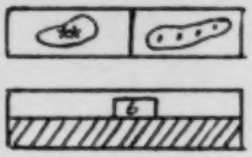
**Concept:** Joining Sets

**Stage of Development:** Semiabstract

**Equipment:** Individual "show-me-boards", cards showing numerals, flash cards showing two sets to be joined

**Activity:** Each child is given a deck of cards showing numerals. The teacher holds up a flash card for all to see. The children are to select from their decks, the card with the correct numeral for joining the sets.

(E.g.,



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**Stage of Development:** Semiabstract

**Equipment:** Large flannel board, "show-me-boards", cards showing numerals, flannel objects, magnetic board, magnetic objects, chalkboard

**Activity:** Two sets of characters are placed on the flannel board, chalkboard, or magnetic board. The children must select the cardinal number of the combined sets from their cards and place it in their "show-me-boards", as described in the preceding activity.

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NOTE: The section on joining sets of the skill sheet is subdivided into the categories (1-6), (1-12), and (1-18). Adapt the activities of this section for these various levels of instruction.

# ADDITION FACTS

## Introduction

Learning addition begins by having the students handle objects, forming sets that later will be replaced with numbers. They first identify the number of objects in the sets that are to be joined, and then the number of the set that is formed. Next, they are introduced to the words "plus" and "equals", and finally to the mathematical symbols + and =, and the equation form for addition.

Basic addition facts involve the renaming of two addends less than ten. All addition combinations can be derived from the basic facts. There are 100 different addition combinations involving the numbers zero through nine. These facts may be reduced to 45 different combinations, through the application of two ideas.

First, the 19 facts involving zero as an addend need not be learned separately, since they are based on the identity principle of addition. Once the student realizes that zero added to any number results in that same number, there is no need to treat each fact involving zero as an addend, as a new idea.

Second, the use of the identity element leaves 81 separate facts to be learned. Of these, 72 can be reduced to 36 different facts by application of the commutative principle. (E.g., if the student learns 3 + 2 = 5, he does not have to learn 2 + 3 = 5 as a new fact.) These 36 facts, plus the nine "doubles" facts (e.g., 2 + 2 = 4, 3 + 3 = 6, etc.), form 45 separate basic addition facts to be learned.

Since knowing the basic facts is an absolute necessity for proficiency in mathematics, it is important to stress reinforcement until knowledge of the facts is achieved. This section is full of games and contests whereby the child is given ample opportunities to practice these basic facts.

Culminating the study of addition facts is the concept of conservation of numbers meaning there are several addition facts which have the same sum. This is often referred to as "number families".

Teachers may also write experience charts with children dictating story problems. Story problems may be duplicated for all children.

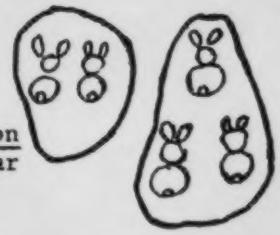
NOTE: If a child demonstrates an understanding of the + and = symbols when doing the activities on pages 100-102, record this as mastery for the (+) symbol on his skill sheet.

**Concept:** "Plus"

**Stage of Development:** All stages

**Equipment:** Overhead projector, magnetic board, flannel board, chalkboard, counting objects (discs, cubes, etc.), or objects to be used on one of these teaching aids

**Activity:** Using one of the above teaching aids, form two sets that are to be joined, such as, The students must express the joining of these sets with a mathematical phrase. For the above sets, it would be 2 + 3. They should use only verbalization at this stage, and this particular activity should be used over a period of several days.



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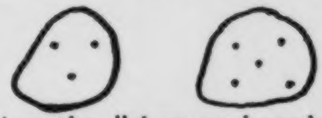
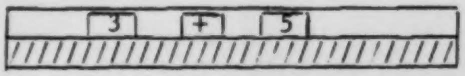
**Stage of Development:** All stages

**Equipment:** Same as in the preceding activity but include "show-me-boards" and cards showing numerals and the + symbol

**Activity:** Each child is given a deck of cards showing numerals and the plus (+) symbol. Each one will form two sets that are to be joined, using one of the above teaching aids. The two sets might look like this: The child is required to express, mathematically, the joining of the sets, using the "show-me-boards", cards showing numerals, and the card showing the (+) symbol. For the above example the following would be necessary:





This can be repeated using different aids and sets. The activity should be extended more than one day.

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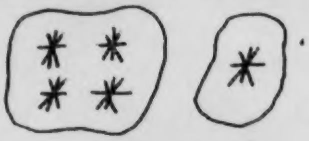
**Stage of Development:** All stages

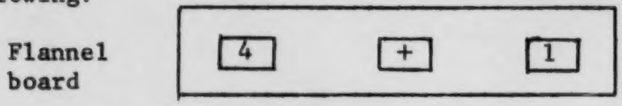
**Equipment:** Overhead projector, magnetic board, flannel board, chalkboard, counting objects (discs, cubes, etc.), or objects for the above teaching aids, individual flannel boards, cards showing numerals and the plus (+) symbol (fixed with velour paper on the backs so they will stick to the flannel boards)

**Activity:** The teacher forms two sets that are to be joined, using one of the above teaching aids. For example,



The children use a mathmatical expression to show the joining of the sets. This is placed on the individual boards.

For the above example, the child should form the following:



Several days should be used for this lesson.

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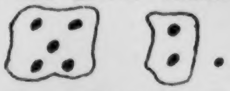
**Stage of Development:** All stages

**Equipment:** Overhead projector, magnetic board, flannel board or chalkboard, objects for these aids, individual chalkboards

**Activity:** Two sets are formed on one of the above teaching aids. For example,



The children will express, mathematically, the joining of these two sets, using the individual chalkboards to record their responses. For the above example:

5 + 2
Individual chalkboard

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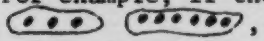
**Concept:** Number sentence

**Stage of Development:** Semiconcrete and semiabstract

**Equipment:** Overhead projector, magnetic board, flannel board or chalkboard, objects for these aids, flash cards showing two sets, or large dominos showing two sets

**Activity:** The two sets that are to be joined are exposed to the class. The children are to respond, verbally, by giving the number sentence associated with the joining of the two sets. For example, if the following two sets were exposed, , the children would say, "Three plus six equals nine."

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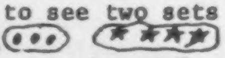
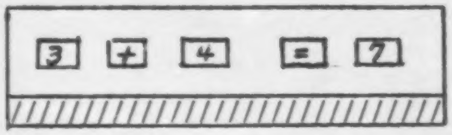
**Stage of Development:** Semiconcrete and semiabstract

**Equipment:** Overhead projector, magnetic board, flannel board or chalkboard, shapes for these aids, flash cards showing two sets, or large dominos showing two sets, "show-me'boards", and cards

**Activity:** The children are allowed to see two sets that are going to be joined (e.g.,  ). They are to use their "show-me-boards" and cards to form the number sentence associated with the joining of these sets. For the above example, the children would show



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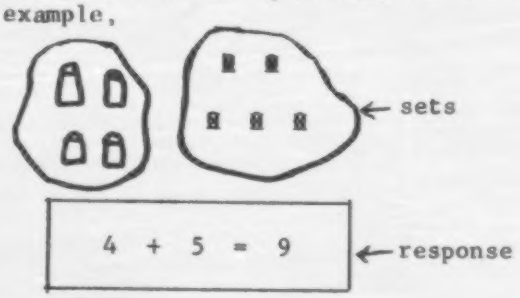
Concept: Number sentence

Stage of Development: Semiabstract

Equipment: Any teaching aid whereby two sets can be formed, chalkboards

Activity: The class sees two sets that are to be joined. The children must then write the correct number sentence associated with the joining of these sets, using the symbols plus (+) and equals (=) as well as the numerals. For example,



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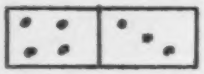
Stage of Development: Semiabstract

Equipment: Chalkboards

Activity: The children are instructed to make dominos (e.g.,  ). When they have finished, the children will come to the front of the room, expose their domino to the class, and tell the domino's story. For the above example, a child would respond with this sentence, "Four plus three equals seven."

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Concept: Joining sets - verbal

Equipment: None

Activity: The teacher makes a verbal statement, such as the following: "I would like to join together a set of two dogs and a set of five dogs. Tell me the number sentence associated with the joining of these two sets." The child would reply, "Two dogs plus five dogs equals seven dogs."

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## CLASSROOM ACTIVITIES

Concept: Addition facts

Stage of Development: Semiabstract

Equipment: Flash cards showing addition problems

Activity: * The children are arranged in a semicircle. A card is then flashed to a certain child. If the child answers the problem correctly, he may keep the card. The child with the most cards at the end of a stated time is declared the winner.

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Stage of Development: Semiabstract

Equipment: "Show-me-boards", cards showing numerals, plus (+) symbol and equals (=) symbol

Activity: Each child is given a "show-me-board" and a deck of cards. The children are told to form a number sentence whose answer is a certain number, and then put it in their "show-me-boards". (E.g., "Make a number sentence whose answer is six.") The children then hold their "show-me-boards" in the air for the teacher's observation. The possible answers for the above example would be: 0 + 6 = 6, 1 + 5 = 6, 2 + 4 = 6, 3 + 3 = 6, 4 + 2 = 6, 5 = 1 = 6, 6 + 0 = 6.

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Stage of Development: Semiabstract

Equipment: "Show-me-boards", cards showing numerals, the plus (+) symbol, the equals (=) symbol, flash cards showing addition problems

Activity: * Each child is given a "show-me-board" and a deck of cards. A flash card, showing an addition problem, is exposed to the class. The children must form a number sentence for this problem on their "show-me'boards". They then hold up their "show-me-boards", with the completed sentence, for the teacher's observation.

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**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** * An addition problem is written on the chalkboard. The children copy the problem on their chalkboards and find its solution. Their chalkboards are then held in the air for the teacher's observation.

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**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** The children are asked to write on their chalkboards a number sentence whose answer will be a number stated by the teacher. (E.g., "Form a number sentence whose answer is seven.") The children then hold up their chalkboards for the teacher's observation. Different solutions are recorded on the chalkboard until all combinations are found.

******************************

**Stage of Development:** Semiabstract

**Equipment:** Chalkboard

**Activity:** A large circle, with a small concentric circle inside it, is drawn on the chalkboard. The large circle should be marked with ten equal divisions. These divisions are labeled with the numerals 0, 1, 2, 3,4,5,6,7,8, and 9. The numerals are not placed in correct sequence around the large circle. A number is placed inside the small circle (e.g., four). The children then give the sum of the numeral inside the small circle plus the numerals around the large circle. Recognition should be given to those children who get all the sums correct. Different problems can be formed by changing the numeral inside the small circle.

******************************

**Stage of Development:** Semiabstract

**Equipment:** Large tagboard cart showing various addition problems, beanbag

**Activity:** * The tagboard chart is taped to the floor. The children stand about 10' or more from the chart. They then throw a beanbag onto the chart (from their position of 10' or more). The number problem on which the beanbag lands is written on the chalkboard for the child to answer. Two teams are formed. If a child answers the problem correctly, his team is given one point. The team with the most points at the end of the game will be the winner.

********************************

**Stage of Development:** Semiabstract

**Equipment:** Chalkboard

**Activity:** * Two children are chosen to work at the chalkboard. They are given an addition problem to solve. The first one to get the correct answer is the winner of the first contest. The winner then chooses someone to compete against him in the next contest. A new problem is given and the winner determined. The winner of each contest remains at the board and chooses his next opponent. When the period is over, the grand winner is the person who was involved in the most contests. (Each child may be chosen only one time.)

********************************

**Equipment:** Flash cards showing addition problems

**Activity:** * The flash cards are placed in the chalk tray so all the students can see them. The class is divided into two teams. Two students, one from each team, stand behind a line drawn 10-15 feet from the chalk tray. The teacher then states a number (e.g., ten). The student that is first to walk to the tray and pick up a flash card (whose answer is this number) is allowed one point for his team. For the example above: $\boxed{\begin{array}{r}4\\ \underline{+6}\end{array}}$ $\boxed{\begin{array}{r}3\\ \underline{+7}\end{array}}$ $\boxed{\begin{array}{r}1\\ \underline{+9}\end{array}}$, etc.

The team that has the most points at the end of the contest is declared the winner.

********************************

**Equipment:** Flash cards showing addition problems

**Activity:** The class is divided into two teams. The teams stand in two rows, with the two children in front facing the teacher. A flash card is held up for these front two children to answer. The first one to answer correctly goes to the end of his line. The one that misses sits down. The competition continues with the two children in front until only one row remains. This team is declared the winner.

✱

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NOTE: The Games and Teaching Aids Book is filled with suggestions of devices that can be used to reinforce the addition facts. These games will serve as motivators for the necessary drill associated with learning the addition facts. They make learning the facts enjoyable. Stay with these games and facts until the student has a mastery.

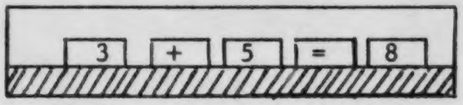
Concept: Conservation of numbers

Stage of Development: Semiabstract and abstract

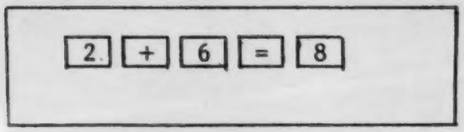
Equipment: Individual flannel boards, "show-me-boards", cards for boards (numerals 1-10 and symbols for plus and equals)

Activity: A number is either shown to the class or verbalized by the teacher. A number sentence, whose sum is this number, is then formed by the students on their "show-me-boards", or on individual flannel boards. (E.g., the teacher exposes "8" to the class, and the children form one of the following number sentences on their boards:
0 + 8 = 8, 1 + 7 = 8, 2 + 6 = 8, 3 + 5 = 8, 4 + 4 = 8,
5 + 3 = 8, 6 + 2 = 8, 7 + 1 = 8, 8 + 0 = 8)

"Show-me-board"



Flannel board



After having formed one sentence, others having the same sum can be formed, until all combinations are found.

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Stage of Development: Semiabstract

Equipment: Individual chalkboards, or pencil and paper

Activity: A number is either shown to the class or verbalized by the teacher. The child writes a number sentence for this number, either as a sum on his chalkboard or on his paper.

******************************

Concept: Number line addition

Stage of Development: Semiabstract

Equipment: Demonstration number line

Activity: The teacher says, "There are four girls and two boys at the reading table. How many students are at the reading table?" (six) Four girls are then shown on the number line. "The spaces between the numbers each represent one girl. We can also call these spaces 'units'. We have represented four girls by using four units on the number line. We can show the addition of the two boys by moving to the right two spaces, or two units. When we use the number line to show addition, we move to the right in the direction of the greater number. Where did we stop? (at six) Are there six students at the reading table? How can you show that this is right?" (The class can count the children.) Other addition problems should be worked on the number line, using units to represent pairs of numbers.

*******************************

Concept: Number line - addition or subtraction

Stage of Development: Concrete

Equipment: Number line on floor

Activity: The number line is placed on the floor. The children are told that to add, one moves to the right, and to subtract, one moves to the left. They can then solve number problems by walking on the number line. (E.g., if the child is given the problem 6 + 4 = ☐, he starts at six and walks to the right four places. He ends up on 10, which is the answer to the problem.) Every child should have an opportunity to solve a problem.

*******************************

Concept: Number line

Stage of Development: Semiabstract

Equipment: Overhead projector, overhead projector pencil, transparencies showing number lines

Activity: A transparency showing a number line is placed on the overhead projector. A child goes to the projector and works out a problem that has been stated (verbally) by the teacher. The child will then draw his solution on the number line. The problems may be addition, subtraction or multiplication, depending upon the maturity of the children.

********************************

Concept: Number line - addition or subtraction

Stage of Development: Semiabstract

Equipment: Individual number lines, large demonstration number line

Activity: The demonstration number line is taped on the chalkboard. An addition or subtraction problem is then written on the board. All of the children are to find the answer, using the individual number lines. When all are finished, one child may come forward and show the class (on the large number line) how he arrived at his answer. This can be repeated with different problems and children. (Grade two children may be given multiplication problems to work.)

********************************

Concept: Number line - addition or subtraction

Stage of Development: Semiabstract

Equipment: Two small number lines, flash cards showing addition or subtraction problems

Activity: The two small number lines are placed side by side on the chalkboard. Two children are called upon to go to these number lines. The teacher flashes a number problem to the children. Each child then tries to be the first one to get the correct answer, using the number lines. The children write their answers on the chalkboard. Two new students may go to the chalkboard to answer the next problem. This can be continued until all children have had two turns.

********************************

**Equipment:** A strip of paper showing a line with 20 equal divisions, flash cards showing addition and subtraction problems, "show-me-boards", cards

**Activity:** The children make a number line from the strip by numbering the divisions. They must begin with zero at the far left division. An addition or subtraction problem is then flashed to the class. The students must use the number line to arrive at the answer to the problem. They then place their answer in the "show-me-boards" and hold them up for the teacher to see.

********************************

**Concept:** Number line - addition or subtraction

**Stage of Development:** Abstract

**Equipment:** Chalkboard

**Activity:** A line is drawn on the chalkboard. One student goes to the chalkboard. He is given an addition or subtraction problem to work. The problem must be written on the board. The child is then told that he has to make a number line out of the line drawn on the board, and work the problem that has been given to him on this number line. (The teacher must emphasize the two important parts of a number line: a zero point, and equal divisions.)

********************************

**Stage of Development:** Abstract

**Equipment:** Paper, pencil

**Activity:** The children are told to make up their own addition or subtraction problems. They are also to make number lines on their sheets of paper. The problems must be worked on the number line.

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## SEPARATING SETS

### Introduction

A child must "reverse" his thinking to think of subtraction as the opposite of addition. Once the ability to think of subtraction as the opposite of addition is established, the child has little difficulty in learning the subtraction facts.

Sets of objects on the flannel board can be used to present examples of set removal. For example, the teacher can place four cutouts of oranges on the flannel board and tell the class he is going to remove two. The class is then asked how many will be left out of the four that were in the original set. The students should determine that the teacher started with a set of four, and that he removed, or separated, a set of two from the original set, and now there is a set of two remaining. The words, "removing" and "separating", should be used as frequently as possible so that the class becomes familiar with their meaning. Although the subtraction operation is not actually being taught at this time, a basis for its introduction is being provided.

# CLASSROOM ACTIVITIES

Concept: Separating sets

Stage of Development: Concrete

Equipment: Tray, counting objects (cubes, beads, discs, sticks, etc.)

Activity: Several objects are placed on the tray so that all the children can see them. The children are questioned concerning the cardinal number of the set. Some of the objects are then removed. The children are to discover how many objects remain. They should give a verbal description of what has taken place. (E.g., "Seven blocks take away three blocks leaves four blocks.") This can be repeated, using the same number of objects, but taking away a different number.

(Note: At this stage the teacher should use the phrase "take away" instead of "minus" or "subtract". Use "leaves" instead of "equals".)

*******************************

Stage of Development: Concrete

Equipment: Set of books or other classroom items

Activity: The class is shown a set of ten books and then one is taken away. The children are asked, "How many are left?" Someone will say, "Ten books take away one book leaves nine books." This procedure can be continued until all separations for ten have been made. The teacher proceeds in the same manner with sets of nine, eight, seven, and six members.

*******************************

Concept: Separating sets

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel objects, piece of yarn

Activity: The yarn is placed on the flannel board so a closed figure is formed. Several objects are placed inside the figure. The children then state the number of elements in the set. Some of the elements are removed from the set. A child can then state the cardinal number of the remaining set.

*******************************

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic shapes, chalk

**Activity:** The teacher draws a closed figure with the chalk and repeats the preceding activity, using the magnetic board and shapes.

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**Concept:** Separating sets

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** The children will form a set of triangles on their chalkboards. For example, the teacher may say, "Draw six triangles on your chalkboard." The children are then told to cross out a certain number of these triangles and to determine the number of remaining triangles. The teacher then says, "Draw an "x" on two of the triangles. Now how many do you have left?" The children must give a verbal description of what has taken place. (They might say, for example, "Six triangles cross out two triangles leaves four triangles.")

********************************

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, objects, pennies

**Activity:** Several pennies, or objects, are placed on the overhead projector. The student is required to state the cardinal number of the set. Some of the elements are then removed, thereby making a smaller set. One child can give the cardinal number of this smaller set. Another one should give a verbal description of what has taken place.

********************************

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, overhead projector pen, acetate roll

**Activity:** A set of objects is drawn on the acetate roll. The teacher then makes up a story which will warrant the removal of some of the elements of the set. The children then determine the number remaining. They must give a verbal description of what has taken place. (For example, the teacher draws six rabbits on the overhead projector, and then crosses out two - saying they hopped away. Four rabbits are left. The verbal statement made by the child would be something similar to the following: "If you had six rabbits and two hopped off you would have four left.")

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| | |
|---|---|
| Stage of Development: | Semiabstract |
| Equipment: | Overhead projector, prepared transparencies |
| Activity: | Several transparencies, showing different sets, are prepared in advance. A transparency is then placed on the overhead projector. One child states the cardinal number of the set. Next, some of the elements of the set are either crossed out or covered, so that the image to the screen is blocked. These will be the elements removed from the set. A child then makes a verbal statement as to what has taken place. |

******************************

| | |
|---|---|
| Stage of Development: | Verbal |
| Equipment: | None |
| Activity: | The children are to look for sets that can be separated. For example: |

1. "There were ten students playing ball. Three went to get a drink of water. How many stayed?"
2. "John had six candy bars. He ate one. How many does he have left?"
3. "Susan had eight pieces of candy, but she lost three of them. How many pieces of candy does she have left?"
4. "Fred caught seven fish, but three of them flopped back into the lake. How many did he have left?"

******************************

NOTE: Separating Sets is subdivided on the skill sheet into the categories (1-6), (1-12), and (1-18). You may want to adapt the activities of this section to these various levels for your classroom instruction.

# SUBTRACTION FACTS

## Introduction

Learning subtraction begins by having the students handle objects, forming sets that later will be replaced with numbers. They first identify the number of objects in a set, then the number associated with a portion of the set that is to be taken away, and, finally, the number of objects remaining in the set. Then they are introduced to the words "minus" and "equals", later to the mathematical symbols - and =, and last to the algorism for subtraction.

Subtraction can be taught by using two different methods. One method is renaming (once called "borrowing"), and the other method is by equal additions.
How can this example be done? 91  Regrouping is used if it is done this way:
-24
67
11 - 4 = 7 (ones); 8 - 2 = 6 (tens). If a person asks himself, "What number added to 24 equals 91?" he is using the equal addition method.

Since knowing the basic facts is an absolute necessity for proficiency in mathematics, it is important to stress reinforcement until knowledge of the facts is achieved. The section on subtraction concepts is full of games and activities which will reinforce the basic subtraction facts.

The equipment used to reinforce the addition facts may also be used to reinforce the subtraction facts. All that needs to be done is to change the addition problems to subtraction problems.

NOTE: If a child demonstrates an understanding of the (-) symbol when doing the activities on page 114, record this as mastery for the (-) symbol on his skill sheet.

**Concept:** "Minus"

**Stage of Development:** Verbal

**Equipment:** Some method of forming a set (counting objects, flannel board, magnetic board, overhead projector, flash cards)

**Activity:** A set is formed by using one of the methods described above. Some of the elements are then removed from the set. The children should verbalize what has taken place, using the term, "minus" at this time. (e.g., Seven objects are placed on the magnetic board, four of them are removed. The child's response should be, "seven minus four".

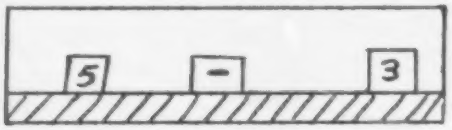
******************************

**Concept:** "Minus"

**Stage of Development:** Semiabstract

**Equipment:** "Show-me-boards", cards showing numerals and the minus symbol, some method of forming a set (counting objects, magnetic board, flannel board, overhead projector, flash cards)

**Activity:** A set of objects is formed for all to see. Part of the objects are removed. The children are asked to show the minus symbol card. (For example, the teacher forms a set of five blocks; then removes three blocks. The child will fix his "show-me-board" as in the following illustration:



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**Concept:** "Minus"

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards, some method of forming a set (counting objects, magnetic board, flannel board, overhead projector, flash cards)

**Activity:** A set is formed, using one of the suggested methods. Part of the elements are taken away from the set. The children then write on their chalkboards, using numerals and the minus symbol, what has taken place. (For example, seven objects are placed on the flannel board. Three are removed. The children will then write on their chalkboards 7 - 3.)

******************************

**Concept:** Number sentence

**Stage of Development:** Verbal

**Equipment:** Some method of forming a set (counting objects, flannel board, magnetic board, overhead projector, flash cards)

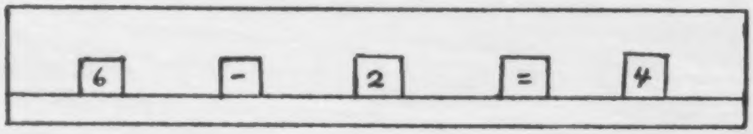
**Activity:** A set is formed by using one of the methods described above. Some of the elements are then removed from the set. The children should verbalize what has taken place, using the terms, "minus" and "equals", at this time. (E.g., seven objects are placed on the magnetic board. Four of them are removed. The child's response should be, "Seven minus four equals three.")

******************************

**Stage of Development:** Semiabstract

**Equipment:** "Show-me-boards", cards showing numerals and the minus (-) symbol, some method of forming a set (counting objects, magnetic board, flannel board, overhead porjector, flash cards)

**Activity:** A set of objects is formed for all to see. Part of the objects are then taken away. The children are asked to form a number sentence on their "show-me-boards" that represents what has taken place. The boards are held in the air for all to see. (For example, the teacher forms a set of six blocks; then removes two blocks. The child will fix his "show-me-board" as in the following illustration:



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**Stage of Development:** Semiabstract

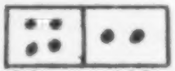
**Equipment:** Individual chalkboards, some method of forming a set (counting objects, magnetic board, flannel board, overhead projector, flash cards)

**Activity:** A set is formed, using one of the above suggested methods. Part of the elements are taken away from the set. The children then write a number sentence on their chalkboards that represents what has taken place. (For example, eight objects are placed on the flannel board. Three of them are removed. The children will write on their chalkboards 8 - 3 = 5.)

******************************

**Stage of Development:** Semiabstract

**Equipment:** Chalkboards

**Activity:** The children are instructed to make dominoes (e.g.,  ). When they have finished, the children will come to the front of the room, expose their domino to the class, cover one side with their hand, and tell the domino's number story. For the above example, a child would respond with this sentence, "Six minus two equals four".

******************************

**Concept:** Subtraction - Number Sentences

**Stage of Development:** Verbal

**Equipment:** None

**Activity:** The teacher makes a verbal statement such as the following: "I had five dogs but three of them ran away". Tell me the number sentence associated with this story. The child would reply, "Five dogs minus three dogs equals two dogs".

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WHEN YOU HAVE REACHED THIS POINT IN DEVELOPING SUBTRACTION FACTS, PLEASE REFER TO THE STARRED (✱) ACTIVITIES IN ADDITION FACTS. ADAPT THESE ACTIVITIES BY USING SUBTRACTION PROBLEMS IN PLACE OF ADDITION PROBLEMS.

**NOTE:** It is essential that students know the subtraction facts. Give your students plenty of practice time to reinforce their understanding of this basic computational skill.

**Concept:** Number line - subtraction

**Stage of Development:** Concrete

**Equipment:** Large number line on floor

**Activity:** The number line is taped to the floor. Someone then stands on a certain number (e.g., six). This student states the number and then walks backward two places (thereby getting a smaller number). The child is then required to state the number he is now standing on and describe what has taken place. "I started on eight, walked backward two places, and now I am on six." This activity is repeated with different children and different instructions.

********************************

**Concept:** Number line subtraction

**Stage of Development:** Semiabstract

**Equipment:** Demonstration number line, pencils

**Activity:** The teacher begins by saying, "Cecilia had six pencils and gave two to Richard. How many did she have left? We can show the six pencils on the number line by moving to the right six spaces from zero and marking a six unit segment. (This should be demonstrated on the number line.) To show the number of pencils she gave away, we move two spaces to the left, cutting a two unit segment from the six unit segment. When we use the number line to show subtraction, we move to the left, in the direction of the lesser number. Where did we stop? (at four) The other segment is a segment of four units. Is four the number of pencils she had left? (yes) How can we show that this is correct?" (Pencils can be used to show that this is the correct answer.)

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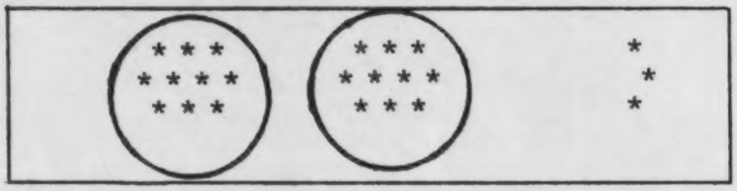
NOTE: See the last few pages of the section on addition facts for more activities on using the number line to solve subtraction problems. Adapt the activities by using subtraction facts for the problems.

# PLACE VALUE

## Introduction



The above figure shows two groups of ten with three stars left over. The importance of place value is evident when one attempts to write a numeral for this number of stars. Instead of writing "two groups of ten and three more," one can simply write 23 and agree that the numeral in the "second place" (2 in this case) is to represent groups of ten.

In a place-value system with base ten, only ten symbols are needed to represent any number. This numeration system uses the digits 0 through 9.

While it is true that each of the digits, 0 through 9, indicates exactly one whole number when it stands alone, the very basis of our place-value system is that these digits may indicate many different numbers when used together. Thus, in the numeral 6,586,624 the first 6, starting from the left, indicates 6 million; the second 6 indicates 6 thousand; and the third 6 indicates 6 hundred.

At the heart of any place-value system is the agreement upon the type of grouping involved. The conventional grouping is by tens, which, it is theorized, may be related to the number of fingers on our two hands.

It is important to provide a careful development of the basic grouping and positional ideas involved in a place-value scheme. An understanding of these ideas is vital to future success (in gaining an insight into the relative size of large numbers and in computing).

NOTE: A firm understanding of the concept of place value is needed before a child is introduced to more than one digit addition, subtraction, multiplication and division problems. Stay with the concept until students have mastery.

**Concept:** Place value

**Stage of Development:** Concrete

**Equipment:** Hangers, boxes, plastic bags, paper plates, etc. Objects: clothespins, quiet counters, teddy bear counters, blocks, paper clips, cereal, etc.

**Activity:** Children chose containers and objects and make sets up to ten, verbalizing the number of sets and the number in each set. Finally, teacher asks children to make sets of ten emphasizing that 10 is a special number.

+ + + + + + + + + + + + + + +

**Concept:** Place value

**Stage of Development:** Concrete

**Equipment:** Bundles of sticks, single sticks, rubber bands, place value boards

**Activity:** Children receive sticks, count them and using the rubber band, put them into sets of 10. When complete, children then count by 10's to see how many each have. Using the place value boards, children are directed to show numerals using bundles of ten and single sticks.

12" x 9"

| Tens | Ones |
|---|---|
|  |  |

+ + + + + + + + + + + + + + +

**Concept:** Place value

**Stage of Development:** Concrete

**Equipment:** Cups, number line, sticks

**Activity:** A number line is placed on the desk or chalk tray so that everyone in the class can see it. A cup is then placed above each of the points, zero through ten. One stick is put in the first ones cup. The teacher will then ask, "How many members has this set of sticks?" The answer should be one. The child then writes "1" on the number line. One stick is placed in each cup until ten is reached. The children should understand that when they get to ten ones, they will have one set of ten or one ten.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Bag of lima beans, tongue depressors, glue

Activity: Give children a handful of lima beans. Ask children to count their beans. As activity progresses, discuss with children any problems they have encountered with their counting task (losing track, beans rolling, etc.). Lead children to "discover" an easier way of counting beans by putting them in groups of ten. After children have separated their beans into sets of ten and counted them, distribute tongue depressors and glue. Instruct children to glue ten beans on each depressor.

A follow-up activity for using these "bean-sticks" is: Teacher writes a numeral on the chalkboard. The child must then select enough "bean-sticks" and loose beans to equal the numeral on the chalkboard.

+ + + + + + + + + + + + + + + +

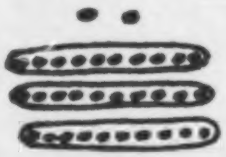
Concept: Place value

Stage of Development: Concrete

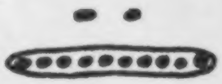
Equipment: Beans, bean sticks and place value boards

Activity: Children are directed to form sets of tens and ones using single beans and bean sticks.

Example: Show me 32,



Show me 12,



+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Bean sticks, chalkboard, single beans

Activity: A child selects a number of bean sticks and single beans. The children at their seats write the numeral representing the total number of beans shown on their chalkboards.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Bean sticks, chalkboard, single beans

Activity: A child chooses a number of bean sticks and single beans. He then chooses another child to write the numeral representing the total number of beans shown on the chalkboard.

+ + + + + + + + + + + + + + +

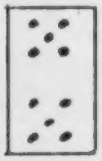
Concept: Place value

Stage of Development: Concrete

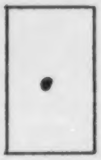
Equipment: Dot cards showing tens and ones and place value boards

Activity: Children form sets of tens and ones using dot cards and place value boards. Example: Show me 53. Show me 14.

2" x 4" - 10 per child



20 per child



+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Set of units and tens rods from Cuisennaire kit

Activity: Using place value boards, children show designated sets of tens and ones. Verbalization between teacher and child emphasizes that it takes ten ones rods to make one ten rod.

+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Diennes blocks and place value boards

Activity: Children use ones and flats to show designated sets of tens and ones

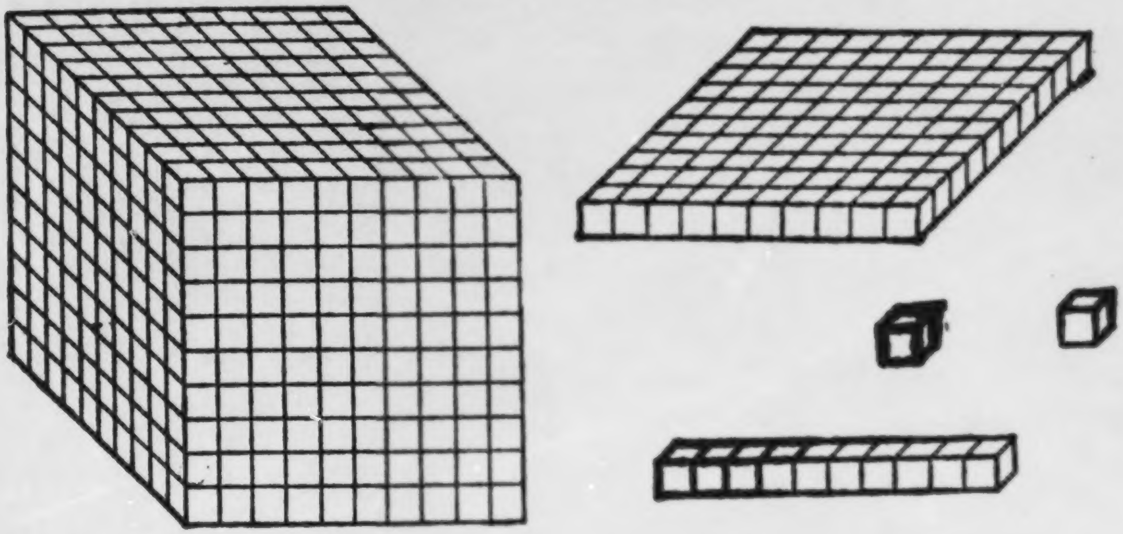
+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Stamp of Diennes blocks on tagboard - place value boards

Activity: Children show a designated amount of tens and ones using stamped cards on place value boards.



+ + + + + + + + + + + + + + +

Stage of Development: Concrete

Equipment: Counting sticks, rubber bands, containers marked "tens" and "ones"

Activity: The definition of a set should be discussed with the children. This concept can be demonstrated with several objects placed in a container. The children will determine the cardinal number of each set. The teacher then holds up four sticks for all to see. The children will describe the number of sticks that he is holding. Their usual response will be four sticks. It should be explained that another way to describe the number of sticks would be "four ones". A different number of sticks is then held up. The children describe the number of this set as so many ones. Ten sticks are held up for them to see. They should describe this set as "ten ones". A bundle of these ten sticks can be made by placing a rubber band around them. The teacher will now need to explain to the children that instead of having "ten ones" they have "a set of ten sticks" or simply "one ten". Three bundles of ten sticks are held up next and the children tell how many tens the teacher has in his hand (three tens). Bundles of ten sticks are then held up in the teacher's left hand (e.g., three) while single sticks are held in the right hand (e.g., five). The children describe the number of sticks the teacher is holding by telling how many "tens" and "ones" are shown (3 tens and 5 ones).

The children are now shown the containers marked "tens" and "ones". Several bundles of ten sticks are placed in the tens container and single sticks are placed in the ones container. Each time this is done the children should tell the number of "tens" and the number of "ones" to represent the number of sticks shown.

+ + + + + + + + + + + + + + +

**Stage of Development:** Concrete

**Equipment:** Counting sticks, rubber bands, containers marked "tens" and "ones", chart for tens and ones, cards for the chart, chalkboard

**Activity:** This activity is a continuation of the one before it. Bundles of sticks and single sticks are placed in their containers and the child shows the number of sticks on the chart, by selecting the proper cards. The children can be asked if they know of a shorter way of describing the number of sticks shown on the chart. Using the example in the preceding activity, it is hoped that they would say, "the number thirty-four". This number should be written on the chalkboard. The entire process can be repeated with other examples. This process can be reversed by placing a two-digit number on the board. The children must then fill the chart with the correct cards, and the boxes with the correct number of bundles and singles of sticks.

+ + + + + + + + + + + + + + +

**Stage of Development:** Concrete

**Equipment:** Numeral cards, bundles of ten sticks, single sticks

**Activity:** Two sets of 5" x 7" numeral cards (0-9) are laid on the teacher's desk so that any card may be easily found by a student. A student then comes to the front of the class. The teacher says to the class, while putting two bundles of sticks in the student's left hand, "I put some bundles of ten sticks in his left hand and no single sticks in his right hand. How many bundles of ten does he have?" (answer, two) "How many single sticks does he have?" (answer, zero) "What is the number? (answer, twenty) Another student may come to the desk and show the number with a numeral card. The class should read the numeral both as two tens and zero ones, and as twenty. Bundles of tens and single sticks are then added to the student's set, and other students come to the desk and pick the appropriate numeral cards for the number of sticks held by the student. This process should be repeated for many numbers from one through one hundred.

+ + + + + + + + + + + + + + + +

**Stage of Development:** Concrete

**Equipment:** Place-value device, individual chalkboards

**Activity:** Several rods of ten beads and several single beads are placed on the place-value device (e.g., three rods of ten and six singles). The children then write on their chalkboards the number that represents the number of beads on the place-value device. The number would be 36 for the above example. This can be repeated by placing a different number of beads on the device.

An alternate activity would be to write a two-digit number on the chalkboard and ask the children to represent the number on the place-value device, using beads. (e.g., for the number 63 the child would select six rods of ten beads and three single beads.)

+ + + + + + + + + + + + + + + +

Stage of Development: Concrete

Equipment: String, paper clips, numeral cards

Activity: A string is stretched across a corner of the room (or any other appropriate place). The teacher places 100 paper clips on his desk. A child then counts out a set of ten. (It should be emphasized that ten ones is one set of ten, or one ten.) The child joins the ten clips together to form one string of clips. Another student takes that set of ten clips and hangs them on the string. The teacher then holds up one of the numeral cards showing a two-digit number (e.g., 43). Someone is asked to represent this number, using paper clips and three single clips. The teacher can also place strings of ten clips and some single clips on the string and ask the children to find the numeral card showing this number of clips.

+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: Money stamp (to make pennies and dimes), place value boards

Activity: Children place dimes and pennies on place value board as directed by the teacher.

+ + + + + + + + + + + + + + +

Note: The above activities involve the use of manipulative objects and mainly the use of verbalization for the number of tens and ones. The following activities will relate the use of numerals with the concrete materials. Use the same instructions as the previous activities.

+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiconcrete

Equipment: Place value board, two digit numeral cards for children

Activity: Children show numerals as teacher places bundles of tens and single sticks in place value chart - activity may be repeated using beans and bean sticks, etc.

+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiconcrete

Equipment: Individual flannel board, tens and ones dot cards, stamp of Diennes blocks, etc.

Activity: Children must show the appropriate numbers of tens and ones to correspond with a numeral selected from a hundreds chart or written on the chalkboard.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiconcrete

Equipment: Magnetic boards, tens and ones dot cards glued to magnets, magnetic Cuisennaire rods, stamp of Diennes blocks

Activity: The children will expose numeral cards to the class while the teacher or students use the available materials to represent the numeral on the magnetic board.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiabstract

Equipment: Overhead projector, bundles of ten toothpicks, plastic ring from canned pop, show me boards

Activity: Teacher places bundles and single toothpicks in rings on the overhead projector while the children put the appropriate numeral cards in their show me boards. Repeat.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiabstract

Equipment: Individual chalkboards, tens and ones manipulative materials

Activity: Use the magnetic board or the overhead projector to show groups of tens and ones. Children will write the appropriate numeral on their chalkboards.

$$\underbrace{\underset{\text{Ex. 2}}{\boxed{\phantom{00}}}\ \text{tens} + \underset{\text{Ex. 3}}{\boxed{\phantom{00}}}\ \text{ones}}_{23}$$

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Overhead transparencies of Diennes blocks, Cuisennaire rods, show-me boards, chalkboards

Activity: Use the transparencies to represent sets of tens and ones on the overhead projector. The children will form corresponding numerals in their show-me boards or chalkboards.

+ + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiabstract

Equipment: Transparencies of dimes and pennies or real coins, overhead, chalkboards or show-me boards.

Activity: Teacher shows sets of tens and ones and children place correct numeral in show-me boards or on chalkboards.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Overhead projector, overhead projector pen

Activity: The following is written on the overhead projector:

$\boxed{\phantom{00}}$ tens + $\boxed{\phantom{00}}$ ones. A two digit number is staged verbally. One child comes forward and fills the boxes with the correct number of tens and ones for the number stated. ($\boxed{6}$ tens + $\boxed{4}$ ones).

As an alternate activity, the teacher might state the number of tens and ones for a number and require the children to write the number on the overhead projector.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiabstract

Equipment: Overhead projector, small transparencies of sets of ten and sets of one or dominoes made by punching holes in tagboard, chalkboards

Activity: A two-digit number is either stated verbally or written on the board. A child is chosen to represent the number on the overhead using sets of tens and sets of one.

An alternate activity would be for the teacher to place a number of sets of tens and sets of one on the overhead. The children are asked to write the numeral representing the total number of dots on their chalkboards.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiabstract

Equipment: Overhead projector, bundles of ten round toothpicks, single toothpicks, chalkboards

Activity: Use the activities described above using toothpicks rather than set transparencies.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Semiabstract

Equipment: Overhead projector, bean sticks, single beans

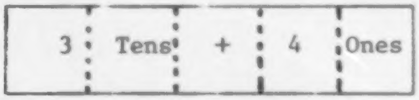
Activity: Use the activities described above using bean sticks and single beans, rather than set transparencies.

+ + + + + + + + + + + + + + + +

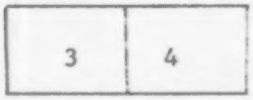
| | |
|---|---|
| Concept: | Place value |
| Stage of Development: | Semiabstract |
| Equipment: | Coffee can math machine |
| Activity 1: | Form two or three digit numeral on the coffee can math machine. Question the children with reference to the number of hundreds, tens and ones of the number shown. |
| Activity 2: | State verbally a certain number of hundreds, tens and ones. Require the children to form a numeral on their coffee can math machines that has the number of hundreds, tens and ones stated. |

+ + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Place value |
| Stage of Development: | Semiabstract |
| Equipment: | Folding place value card |

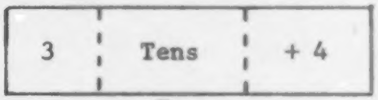


Completely Unfolded

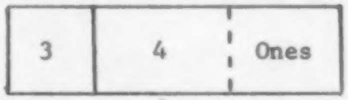


Folded

| | |
|---|---|
| Activity: | Begin with card completely folded, exposing only the two-digit numeral. Ask the children to identify the numeral. Ask the children to state the number of tens and ones that make up the numeral, then show them the correct answers by unfolding the card as shown below. |



Tens



Ones

+ + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Place value - expanded notation |
| Stage of Development: | Semiabstract |
| Equipment: | Chalkboards, numeral cards showing two-digit or three-digit numerals |

Activity: One of the numeral cards is exposed to the class, (e.g. $\boxed{35}$). Ask the children to use expanded notation to represent this numeral on their chalkboards. For the above example, the children would write 35 = 30 + 5. The numeral 642 would be written 600 + 40 + 2.

+ + + + + + + + + + + + + + +

Concept: Place value - expanded notation

Stage of Development: Semi-abstract

Equipment: Chalkboards, cards showing two or three digit numerals written in expanded notation

Activity: Expose one of the cards to the class (e.g. 30 + 7). Require the children to write the numeral for this expansion. For the above example 37. The card showing 500 + 40 + 3 would yield the numeral 543.

+ + + + + + + + + + + + + + +

## THE NUMBERS 100-999

To develop the concept of place value for the numbers 100-999, the teacher must establish the groupings of hundreds, tens, and ones. When using sticks or toothpicks, put ten bundles of ten sticks together to represent a set of one hundred. When using the rectangular strips showing sets of ten dots, bundle ten rectangles together to represent one hundred. When using beans, glue ten bean sticks onto a sheet of tagboard to represent a set of one hundred beans, or bundle ten bean sticks together. When using money, use a dollar bill to represent one hundred.

An excellent commercial product that can be used to represent place value is a "Chip Trading Kit." This kit utilizes different groupings in the place-value system. (e.g., Red chip represents hundreds, green chip represents tens, yellow chip represents ones, etc. Therefore 3 red chips, 2 green chips and 4 yellow chips would represent the number 324.) You can make a kit similar to the "Chip Trading Kit" using different colored blocks, discs, pieces of wood, bottle caps, etc.

NOTE! To develop the concept of place value for three-digit numerals, use the activities described in the preceeding session. The only difference being the use of groups of 100 (sticks, toothpicks, dot cards, beans, money, etc.) to represent the three digit numerals.

## THE NUMBERS 1,000-1,000,000,000

It is very difficult, if not impossible, to use manipulative materials to physically represent numbers in the range 1,000 to 1,000,000,000. It is possible to represent these numbers by using different objects or different colors of the same object. To this an agreement has to be reached relative to the place value of each object or color.

e.g. □ - million, ☒ - hundred thousand, ✠ - ten thousand,

△ - thousands, ○ - hundreds, ⊙ - tens, • - ones.

Therefore, □ □ ☒ ✠ ✠ △ △ △ ○ ○ ○ ⊙ ⊙ • • •

would represent the number 2,123,323.

Red - ten thousand, Green - thousand, White - hundred, Orange - tens, Black - ones

then, RRR GGGG WW OOO BBBBB would represent the number 34,235.

When working with the larger numbers, let your students help devise ways of representing the different digits. Have fun!

Concept: Place value, hundreds, tens and ones

Stage of Development: Concrete

Materials: Cans marked 100, cans marked 10 and individual objects to show place value: 3 x 5 index cards

Activities: You may want to give the students exercises similar to this: 23 ones = $\boxed{\quad\text{tens}}$ + $\boxed{\quad\text{ones}}$. The initial exercises can be left simple, so that the students will be able to use manipulative aids for help, if necessary. You may use 10 and 100 cans, tongue depressors in bundles, money, or other objects of your choosing to demonstrate place value.

+ + + + + + + + + + + + + + + +

Concept: Place Value, thousands, hundreds, tens and ones

Stage of Development: Concrete

Materials: Four paper bags, strips of paper

Activity: Inside each paper bag are strips of paper showing numerals that are multiples of thousands, hundreds, tens or ones. For example, the thousands bag would have 1,000, 2,000, 3,000, etc. A game can be played in which a player draws one strip of paper from each of the bags and forms a numeral from summing up the numbers from his drawing. The greatest number wins.

As a possible variation, you might provide four spinners for forming the numerals. One spinner would show thousands, one would show hundreds, another ten and another the ones.

+ + + + + + + + + + + + + + + +

Concept: Place value, thousands, hundreds, tens and ones

Stage of Development: Concrete

Materials: Cubes with numerals 1 - 6 printed on them

Activities: You may ask each student to roll two cubes and arrange them to make up the greatest number possible using the numerals as digits. Then ask him to rearrange them to make another numeral. A discussion might result as to how he knows which number is greater. Perhaps the students can come up with a rule for choosing the greater number.

Next have the student roll three cubes and arrange them to represent numbers, first the largest, then the next largest and so on, ending with the least. Students could record their findings and compare notes. Some students may want to roll four cubes and arrange the 24 possible numerals from greatest to least.

+ + + + + + + + + + + + + + + +

Concept: Place value

Stage of Development: Concrete

Equipment: A set of ten cards labeled 0 - 9 for each team

Activities: You may want to organize the class into teams of four or more students to play the "greatest number" game. Two teams compete against each other. Each team has a set of the 0 - 9 cards in a paper bag. Each team takes a turn choosing at random a card at a time from the bag to build a (e.g. six digit) numeral. The object of the game is to build a numeral that names the largest possible number using the digits drawn. The result of each drawing is recorded in whatever position of a place-value chart the players choose. The card is then returned to the bag, all the cards are mixed and the next drawing is made by another team member.

TEAM A

| | | | | | |
|---|---|---|---|---|---|
| | | | | 3 | 0 |

TEAM B

| | | | | | |
|---|---|---|---|---|---|
| 9 | | | | | 0 |

After six draws the team that has recorded its draws so as to name the larger number wins the game.

+ + + + + + + + + + + + + + + +

Stage of Development: Classroom game

Equipment: 3" x 5" index cards

Activity: Make a deck of playing cards with index cards for place-value drill. A deck consists of ten pairs of matching cards and one odd card, which you can call the "old maid" if you wish. Write a numeral like 18 ones or 36 tens on one of the two matching cards.

The mates for these two cards would be 1 ten + 8 ones and 3 hundreds + 6 tens respectively. Three students can play. The dealer gives seven cards to himself and the other two players. Play moves to the left. The player to the left of the dealer begins by picking a card from the hand of the player to his left. A player places a set of matching cards on the table in front of himself whenever he has a match. The object of the game is not to have the odd card at the end.

+ + + + + + + + + + + + + + +

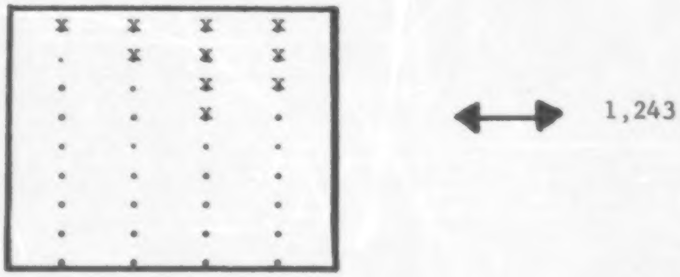
Stage of Development: Concrete

Materials: Sheet of pegboard, labels, golf tees

Activities: Provide the students with a sheet of pegboard on which they may paste the labels for the place-value headings. After putting golf tees (or any kind of peg that will fit the board) in a line under each heading, the students can write the standard numeral represented.

You may leave a column blank sometimes so a zero will appear in their numeral. Students may take turns "pegging" a number while others write the standard form. If you have several peg boards available, students could work together in pairs or small groups.

After students show an understanding of this, they can reverse the process by naming a numeral in standard form and have someone "peg" it on the board. Example:



Concept: Expanded notation

Stage of development: Semiabstract

Equipment: Individual chalkboards

Activity: Being able to write a number in expanded notation will be very important when learning the concept of renaming and when developing the skill of long division.

Write a numeral on the chalkboard (e.g., 5,672). The students write the number in expanded notation on their chalkboards. They should write 5000 + 600 + 70 + 2. Repeat with other numbers. To check for understanding, write the expanded form on the chalkboard and ask the students to write the correct number on their boards.

e.g., you write: 80,000 + 6,000 + 400 + 70 + 3

and they write: 86,473.

+ + + + + + + + + + + + + + +

Stage of development: Semiabstract

Equipment: Overhead projector

Activity: Repeat the above activity but have students write their responses on the overhead projector.

+ + + + + + + + + + + + + + +

# REGROUPING

## Introduction

Regrouping, often called in traditional arithmetic programs "carrying" and "borrowing," has been historically a very difficult process to master. Note the word process in the preceeding sentence; it has been the traditional approach to regrouping as a process rather than a concept which has presented the difficulty. In the contemporary mathematics program, regrouping is presented as a concept first and a process second. It is a very fundamental aspect of the overall structure of mathematics and should be afforded recognition as such. (Many of the problems that children have in multiplication and division, once they get past the basic facts, stem from a lack of mastery of the concept of regrouping rather than a deficiency in multiplication or division.)

The children should be given extensive practice in analyzing the regrouping concept as they progress to the standard algorism applying their understanding to the process. For example, they find the sum of 48 and 23 as follows:

A.

$$\begin{aligned} 48 &= 40 + 8 \\ \underline{+23} &= \underline{20 + 3} \\ &= 60 + 11 \\ &= 60 + (10 + 1) \\ &= (60 + 10) + 1 \\ &= 70 + 1 \\ &= 71 \end{aligned}$$

B.

$$\begin{aligned} 48 &= 40 + 8 \\ \underline{+23} &= \underline{20 + 3} \\ &= 60 + 11 = 70 + 1 = 71 \end{aligned}$$

C.

$$\begin{array}{r} 48 \\ \underline{+23} \\ 71 \end{array}$$

As is the case with most mathematical concepts and processes, regrouping is best understood through the use of concrete materials. Consequently, the concept of regrouping is introduced with manipulative materials. Some possible materials are listed below:

- popsicle sticks
- pop lids (pepsi, 7-up, coke, etc.)
- poker chips
- toothpicks
- coffee stirrers
- gum wrappers ( white - spearmint; green - doublemint; yellow - juicy fruit)
- beansticks

NOTE: This concept is critical for success in computation. Stay with the concept until your students have a thorough understanding of it.

Concept: Regrouping (renaming)

Stage of development: Concrete

Equipment: Toothpicks, popsicle sticks, coffee stirrers

Activity 1:

One way to start the session is by holding up a bundle of ten and asking the students how many there are in the bundle. With the emphasis on place value in textbooks, most students answer ten; if not the sticks may be unbundled and counted. The same procedure is used for introducing the idea that the next larger size is made of ten groups of ten (hundreds).

Several different combinations of bundles, such as 6 tens 2 ones, may be used to help the students learn to recognize the quantity of sticks. A music stand or the chalk tray can be used to hold the different combinations of bundles so they can be more easily seen than when they are placed on the table. A possible sequence of combinations of bundles to follow is:

1. 2 tens 3 ones
2. 4 tens 1 one
3. 3 tens 11 ones
4. 5 tens 2 ones
5. 4 tens 12 ones

Problem 3 is introduced to help the student formulate the idea that a quantity can be expressed different ways and definitely should follow example 2. It is suggested to follow problem 3 with 2 tens 21 ones etc. and record the pattern on the chalk board. This concept of being able to represent a quantity different ways is important in understanding the four basic operations of whole numbers. For example, consider the subtraction example $\begin{array}{r}32\\ \underline{-9}\end{array}$. The decomposition of 32 into $\overset{2}{\cancel{3}}\,12$ is the same as expressing 3 tens 2 ones as 2 tens 12 ones. (Regrouping)

Concept: Regrouping (renaming)

Stage of development: Concrete

Equipment: Toothpicks, popsicle sticks, coffee stirrers

Activity 2: Hand out bundles of tens and single sticks to the students. Give your students a large number of single sticks (e.g., 36) and ask them what the maximum number of bundles of tens that could be formed with these sticks. If they don't know the answer, have them bundle the sticks into tens. Repeat with other groups of single sticks.

Activity 3: A next logical step is to have the students represent the same quantity (3 tens, 6 ones) with different groupings such as 2 tens, 16 ones. The students using the sticks will find different representations for this one quantity. Only examples of two digit numbers should be used.

These important ideas of positional notation and regroupings can be further emphasized by listing the findings of the students. The example, 3 tens, 6 ones, should be placed on the chalkboard and the different groupings as formed by the students should be recorded. For example,

$$\left.\begin{array}{l} \text{3 tens, 6 ones} \\ \text{2 tens, 16 ones} \\ \text{1 ten, 26 ones} \\ \text{0 tens, 36 ones} \end{array}\right\} \quad \text{Regroupings}$$

It might be asked at this time, "If a student has 36 sticks, what is the most number of ones that can be formed?" These questions should be asked after each example until the students understand the concept. Several more examples similar to 3 tens, 6 ones should be presented where the above procedure is followed, that is, recording the results of the students on the board.

+ + + + + + + + + + + + + + +

Worksheet #1 Name _______________

A. Write 8 tens, 2 ones in three different ways.

1. ____ tens ____ ones

2. ____ tens ____ ones

3. ____ tens ____ ones

B. Write 5 tens, 6 ones in five different ways.

1. ____ tens ____ ones

2. ____ tens ____ ones

3. ____ tens ____ ones

4. ____ tens ____ ones

5. ____ tens ____ ones

C. Write 6 tens, 4 ones in six different ways.

1. ____ tens ____ ones

2. ____ tens ____ ones

3. ____ tens ____ ones

4. ____ tens ____ ones

5. ____ tens ____ ones

6. ____ tens ____ ones

D. Write 7 tens, 3 ones in all possible ways.

1. ____ tens ____ ones

2. ____ tens ____ ones

3.

4.

E. ______ = What is the most groups of tens that can be made from 62 sticks?

F. ______ = What is the most groups of ones that can be made from 92 sticks?

G. ______ = 6 tens, 17 ones

H. ______ = 3 tens, 23 ones

I. 39 = 2 tens _____ ones

178

**Activity 4:** Hand out the bundles of ten sticks and the single sticks and review activity 3.

Review may start by having the students find the regroupings of 6 tens, 7 ones such as 5 tens, 17 ones, etc. Again, in presenting examples to the student, it is recommended that the examples be written on the board, then the student should try to work the problem at his desk with the sticks before there is any group discussion. It should be stressed that the different representations are the same value. The figuring of the maximum number of groups can be discussed after each regrouping problem.

A possible start for the introduction of the new material is to consider one of the problems from the preceding worksheet, for example, 39 = 2 tens ____ ones. The students formally have not been exposed to completing place-value sentences. DO NOT treat place-value sentences as new material, as the children have already experienced the problems on the worksheet. The same procedure of presenting the material should be followed as above, that is, the example should be written on the board and then the student should try and work it at his desk before there is any group explanation of the solution.

If the students have trouble with 39 = 2 tens ___ ones, questions should be asked such that they will see the pattern of exchanging a ten gives 10 more ones or in student language, so he has to unbundle a ten to give 10 ones. Students should be encouraged to use the sticks until they demonstrate an understanding of the concept of regrouping.

Several other examples should be presented to reinforce the above concept; for example, 26 = 1 ten ____ ones, 39 = ____ tens, 19 ones, and ____ = 3 tens, 14 ones. It should be noted only examples involving a difference of 1 ten have been formally introduced.

+ + + + + + + + + + + + + + + +

Worksheet #2 Name ______________

A. Write 6 tens, 5 ones in three different ways.

1. ____ tens ____ ones

2. ____ tens ____ ones

3. ____ tens ____ ones

B. Write 9 tens, 3 ones in five different ways.

1. ____ tens ____ ones

2. ____ tens ____ ones

3. ____ tens ____ ones

4. ____ tens ____ ones

5. ____ tens ____ ones

C. 63 = 5 tens ____ ones

D. 52 = 4 tens ____ ones

E. ____ = 3 tens, 16 ones

F. ____ = 5 tens, 13 ones

G. 38 = ____ tens, 18 ones

H. 64 = 4 tens ____ ones

I. 54 = 3 tens ____ ones

J. What is the most groups of tens that may be formed from:

63 sticks? _____ 85 sticks? _____ 74 sticks? _____

K. On the back, represent 1 hundred, 3 tens, 6 ones in several different ways.

Activity 5: At this point in the study of regrouping, it might be well to give the students an opportunity to apply what they have learned by playing a game of place value bingo. A 6" x 6" bingo card may be constructed for each student similar to the illustration below.

| | | |
|---|---|---|
| | | |
| | FREE | |
| | | |

Have the students select their own numbers for their card. Let them choose any numbers between 20 and 29 for the first column, any number between 30 and 39 for the second column, and any number bewteen 40 and 49 for the third column. After the students have recorded their selections, numbers are called in the form of 1 ten 12 ones; 3 tens 17 ones, etc. Give the student counters to cover the numbers called that they have on their boards. The first student to cover three in a row is the winner. This game can be adapted for numbers in the hundreds.

BINGO CHART PATTERN

(Make a ditto and run off enough for your students.)

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Activity 6:

The purpose of the session is to formally introduce the rewriting of a three-digit number. If necessary, a review may start by rewriting two-digit numbers and reworking place-value sentences. It has been found that most children understand these concepts now and not more than one example of each needs to be presented.

If the sticks have not been distributed among the students, this should be done at this time. To employ rewriting three-digit examples in the discussion, the example 1 hundred 3 tens 6 ones on the previous worksheet may be considered. The example should be written on the board and the students given ample time to investigate with the sticks before there is group discussion. Many students by now have developed the pattern for writing a three-digit number different ways and therefore do not rely on the sticks. Even though some students have developed the pattern of the three-digit number, those that have not should be given time to manipulate the sticks. The teacher, when representing the findings of the student on the board, should only represent as many combinations of the three-digit problem as are needed for the student to develop the pattern. If all the combinations are presented, it is quite time consuming. The students do enjoy the activity and it has been found difficult to terminate this activity in the middle of the lesson. Another three-digit example should be introduced at this time.

Give the student worksheet #3 at this time. Notice that three-digit place-value sentences are introduced on the worksheet.

An interesting demonstration for the students is to take ten hundreds and tape them forming a thousand. The students are quite impressed with the magnitude of a thousand as compared to a hundred (ten bundles of ten).

+ + + + + + + + + + + + + + +

Worksheet #3

Name ____________________

A. Write 5 tens, 7 ones in different ways.

1. ____ tens ____ ones
2. ____ tens ____ ones
3. ____ tens ____ ones
4. ____ tens ____ ones
5. ____ tens ____ ones

B. 63 = ____ tens, 13 ones

C. 84 = ____ tens, 25 ones

D. ____ = 4 tens, 23 ones

E. 63 = 4 tens ____ ones

F. 57 = 4 tens ____ ones

G. 33 = 2 tens ____ ones

H. Write 2 hundreds, 3 tens, 5 ones in five different ways.

1. ____ hundreds ____ tens ____ ones
2. ____ hundreds ____ tens ____ ones
3. ____ hundreds ____ tens ____ ones
4. ____ hundreds ____ tens ____ ones
5. ____ hundreds ____ tens ____ ones

I. 234 = 2 hundreds, 2 tens ____ ones

J. 143 = 1 hundred, 3 tens ____ ones

K. 186 = 1 hundred ____ tens, 16 ones

L. What is the most groups of hundreds that may be formed from:

123 sticks? _____ 234 sticks? _____ 567 sticks? _____

M. What is the most groups of tens that may be formed from:

93 sticks? _____ 46 sticks? _____ 234 sticks? _____

Stage of development: Abstract

Equipment: Worksheet

Activity: Complete the worksheet.

A. Write 2 hundreds 2 tens 3 ones in eight different ways.

1. ____ hundreds ____ tens ____ ones
2. ____ hundreds ____ tens ____ ones
3. ____ hundreds ____ tens ____ ones
4. ____ hundreds ____ tens ____ ones
5. ____ hundreds ____ tens ____ ones
6. ____ hundreds ____ tens ____ ones
7. ____ hundreds ____ tens ____ ones
8. ____ hundreds ____ tens ____ ones

Complete the sentences:

B. 59 = 2 tens ____ ones

C. 232 = 2 hundreds ____ tens 12 ones

D. 265 = 1 hundred ____ tens 5 ones

E. 334 = 2 hundreds ____ tens 4 ones

F. What is the most groups of hundreds that may be formed from:

236 sticks? _____ 431 sticks? _____ 94 sticks? _____

G. What is the most groups of tens that may be formed from:

73 sticks? _____ 126 sticks? _____ 231 sticks? _____

H. Write 5 tens 4 ones in three different ways.

1. ____ tens ____ ones
2. ____ tens ____ ones
3. ____ tens ____ ones

Stage of development: Abstract

Equipment: Worksheet

Activity: Complete the worksheet

A. Write 7 tens 4 ones in four different ways.

1. ____ tens ____ ones
2. ____ tens ____ ones
3. ____ tens ____ ones
4. ____ tens ____ ones

B. Write 2 tens 3 ones in three different ways.

1. ____ tens ____ ones
2. ____ tens ____ ones
3. ____ tens ____ ones

Complete the sentences:

C. 75 = 6 tens ____ ones

D. 84 = 6 tens ____ ones

E. 43 = ____ tens 13 ones

F. 37 = 1 ten ____ ones

G. Write 2 hundreds 1 ten 6 ones in five different ways.

1. ____ hundreds ____ tens ____ ones
2. ____ hundreds ____ tens ____ ones
3. ____ hundreds ____ tens ____ ones
4. ____ hundreds ____ tens ____ ones
5. ____ hundreds ____ tens ____ ones

H. 237 = 2 hundreds 2 tens ____ ones

I. 538 = 4 hundreds ____ tens 8 ones

J. 431 = 3 hundreds ____ tens 1 one

K. What is the most groups of tens that may be formed from:

67 sticks? ____ 241 sticks? ____ 317 sticks? ____

186

# ADDITION OF WHOLE NUMBERS

## MISSING ADDEND

### Introduction

The study of missing addends is an important step toward the solution of subtraction problems. All subtraction problems can be written as missing addend problems. (e.g., $9 - 4 = \square$ could be written as $4 + \square = 9$.)

## SUM OF THREE ADDENDS

When working with a problem involving the sum of three addends, the child should remember two important concepts: (1) He can add only two numbers at a time. (2) To add more than two numbers, the addends must be grouped. He may group addends any way he wishes without affecting the sum (e.g., $2 + (3 + 4) = (2 + 3) + 4$). This second concept is called the associative property of addition. When students are working problems involving the sum of three addends, the teacher must allow them ample opportunity to discover that the sum is the same, regardless of the way the addends are grouped.

## TWO AND THREE DIGIT ADDITION

After a child has mastered the basic addition facts, has an understanding of place value, and has an understanding of regrouping, he should be able to add any two whole numbers.

## WORDED PROBLEMS

There are plenty of good worded problems in our adopted text. If you make up worded problems, make them as realistic to the student as possible - the kind of problems the student might encounter.

Concept: Missing addend

Stage of Development: Concrete

Equipment: Rings and things

Activity: Give each child some blocks--not to exceed ten. Give him also a ring made from tagboard. Have children put a given number of blocks on the left of the ring. "Put two blocks on the left side of your empty ring." Then tell them to add enough more inside the ring to make a certain number altogether. "Add enough more to make seven altogether." The children would then put five blocks into the ring and give the appropriate number sentence or explanation. The teacher should be demonstrating the same problem on the magnetic or chalkboard and at the same time asking leading questions such as, "How many more did you add? How many do you add to two to get seven? Two and how many more make seven?"

### POINTS TO REMEMBER ABOUT THE MISSING ADDEND

This concept must be teacher-directed with written work being done as recording of the actual manipulative process. There is no real merit in worksheets on this concept unless children thoroughly understand the process.

+ + + + + + + + + + + + + + +

Stage of Development: Semiconcrete

Equipment: Magnetic board

Activity: Characters are placed on magnetic board for all to see. Teacher shields some of the objects and children verbalize as to missing addend.

Repeat above activity using overhead projector and coins, counters, etc.

+ + + + + + + + + + + + + + +

Concept: Missing addend

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: A frame problem involving a missing addend is written on the chalkboard (e.g., $\boxed{\phantom{0}} + 6 = 8$ or $3 + \boxed{\phantom{0}} = 7$). The children are asked to write the problem on their chalkboards and then find the number for the frame that will make the sentence true. The children then hold their chalkboards in the air for the teacher's observation.

+ + + + + + + + + + + + + + + +

Equipment: "Slide-o-math" cards

Activity: A slide is placed over one of the addends, exposing the rest of the sentence to the children. They are asked to supply the missing addend. To show them the correct answer, the teacher merely moves the slide to the right or left.

(Note: It is possible to let one child work with another child, using the "slide-o-math" cards as a drill activity.)

+ + + + + + + + + + + + + + + +

Concept: Missing addend

Stage of Development: Verbal

Equipment: None

Activity: A number of children stand before the class (5, 6, 8, etc.). The children state how many are there. Then the class hides their eyes while some of the children hide or are removed. The class is then asked, "How many are missing?" or "How many more do we need to make ________?"

+ + + + + + + + + + + + + + + +

**Equipment:** "Dial-a-number" board

**Activity:** The teacher writes a number on the board. This number must be greater than any number on the "dial-a-number" board (e.g., 14 if the board goes to 12). The disc with a hole in it is spun, exposing a number (e.g., six). The children are told that the number the teacher wrote on the board is to represent the answer to an addition problem, and the exposed number is one addend. A verbal response is then given.

+ + + + + + + + + + + + + + +

Concept: Three addends

Stage of Development: Concrete

Equipment: Blocks, rings - make three sets

Activity: Children use blocks and rings to make three sets and verbalize number sentence for the sets. To find the sum, children combine the first two sets and add the third set to the sum of the first two.

+ + + + + + + + + + + + + + +

Concept: Three addends

Stage of Development: Semiconcrete

Equipment: Individual flannel boards, shapes for the flannel boards

Activity: The children are to count out a certain number of flannel shapes to use on their boards (e.g., ten). They are told to make three sets out of these shapes. One possible answer for the above example is:

```
* *    * *    * *
       * *    * *
```

One child then brings his flannel board to the front of the room, shows it to the rest of the class, and writes an addition problem for the joining of the sets on the chalkboard. (The sentence for the example is 2 + 4 + 4 = 10.)

+ + + + + + + + + + + + + + +

Concept: Three addends

Stage of Development: Semiconcrete

Equipment: Magnetic board

Activity: Using magnetic characters, a child separates a set into three subsets on the magnetic board. The other children state the cardinal number for each subset as the teacher writes it. The teacher then puts parenthesis around the first two subsets and the children verbalize the complete number statement. The parentheses are then transferred to the other subsets. Children verbalize the process.

e.g., ( * * * * ) * *
*  *

(3 + 2) + 3 = 8

* * ( * * * * )
*  *

3 + (2 + 3)

+ + + + + + + + + + + + + + +

**Concept:** Three addends

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** The teacher writes a number on the board (e.g., eight). The children are asked to use three numbers in an addition problem. (These numbers must add up to the number on the board.) Some possible answers for the above example are 2 + 2 + 4 = 8, 6 + 1 + 1 = 8, 3 + 3 + 2 = 8, etc. The different answers are written on the board. The beginning number is then changed and the activity is repeated.

+ + + + + + + + + + + + + + +

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** A verbal description of an addition sentence involving three addends is given to the children. They write the problem on their chalkboards and find the sum. When all are finished, they hold up their chalkboards for the teacher's observation.

+ + + + + + + + + + + + + + +

## CLASSROOM ACTIVITIES

Concept: 2 two-digit addition

Stage of Development: Concrete

Equipment: Counting sticks, boxes labeled tens and ones

Activity: Several bundles of ten sticks are made. An addition problem involving 2 two-digit numbers is written on the board. A child comes forward and represents each number by placing one bundle of ten sticks for each ten in the "tens" box and enough single sticks in the "ones" box to represent the ones. Two "tens" and two "ones" boxes are used. To find the sum, the bundles of tens sticks and the number of ones are combined. If the number of ones is greater than ten when they are combined, another bundle of ten is made by taking ten of these ones. This demonstrates the concept of regrouping.

******************************

Stage of Development: Concrete

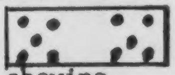
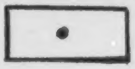
Equipment: Place value device

Activity: An addition problem showing 2 two-digit numbers is written on the chalkboard. A child represents the two numbers on the place-value device, using different-colored beads for each number. He should use a string of ten beads for each ten in the number, and single beads to represent the ones. To find the answer, he adds the number of strings of ten beads and the number of single beads. If there are more than ten single beads, another string of ten must be formed, using ten singles.

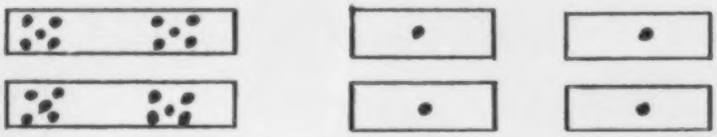
******************************

Concept: 2 two-digit addition

Stage of Development: Semiconcrete

Equipment: Individual flannel boards, rectangular pieces of construction paper showing sets of ten dots , rectangular pieces of construction paper showing sets of one dot 

**Activity:** An addition problem involving 2 two-digit numbers is written on the chalkboard. The children can represent each number by using the pieces of construction paper showing sets of ten and one. For example, the number 24 would be as follows on their flannel boards:



The sum is found by joining the "tens" pieces and the "ones" pieces. If there are more than ten pieces showing one element, the ten pieces showing one element should be traded for one piece showing ten elements. This is the process of regrouping.

********************************

**Concept:** 2 two-digit addition

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** A problem showing the addition of 2 two-digit numbers is written on the chalkboard. The children copy the problem on their chalkboards and compute the answer. When they are finished they must hold their boards in the air for the teacher's observation. More interest can be created by having a contest to see who can finish first.

********************************

**Concept:** 2 two-digit addition

**Stage of Development:** Abstract

**Equipment:** Paper, pencil, "show-me-boards"

**Activity:** A problem showing the addition of 2 two-digit numbers is written on the chalkboard. The children write the problem on their paper and find the sum. They then represent their answers on their "show-me-boards". The boards are held in the air for the teacher's observation.

## CLASSROOM ACTIVITY

Concept: 2 three-digit addition

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: The teacher either writes on the chalkboard, or verbalizes, an addition problem involving 2 three-digit numbers. The children copy the problem on their chalkboards and find the sum. When they have finished they should hold their chalkboards in the air for the teacher's observation. (It is important to use numbers that do not involve regrouping when the children are finding the sum.)

******************************

NOTE: The activities of this section are geared to two digit addends, but may be used for three or four digit addends by using larger numbers.

# SUBTRACTION OF WHOLE NUMBERS

## Introduction

When a child understands the concept of place value and the process of regrouping, and when he knows the basic subtraction facts, he will be able to find the answer to any subtraction problem involving whole numbers. This section concerns all three of the above rudiments of subtraction.

When a child is working a subtraction problem involving two-digit numbers, the teacher must make sure he subtracts the number of ones before he does the tens.

NOTE: When working with three digit numbers in subtraction (no regrouping), use the same activities as you did with two digit numbers. Remember to stress that the children must first subtract the ones, then tens and finally the hundreds. If you want to use manipulative materials, try using toothpicks. A bundle of one hundred toothpicks is not as bulky as a bundle of counting sticks.

Use the worded problems found in the adopted text for the application of multi-digit subtraction. If you make up your own subtraction problems, please make them realistic.

Once the students master the concept of renaming, in order to subtract the numbers, they will no longer need the sticks.

**Concept:** Subtraction of two-digit numbers

**Stage of development:** Concrete

**Equipment:** Toothpicks, popsicle sticks, coffee stirrers

**Activity 1:** Give each student bundles of tens and single sticks. Review the various ways a number of sticks can be regrouped, e.g., 5 tens 4 ones, or 4 tens 14 ones, or 3 tens 24 ones, etc. Do this with several examples.

You may want to establish the concept of subtraction with your students. Usually the student thinks of subtraction as "take away." Write 4 tens 3 ones on the board. Pick up 4 bundles of tens and 3 single sticks and ask one of the students to come up and take 18 sticks from the quantity. A class discussion should develop about how the student will have to take a ten apart before 8 ones can be removed. The procedure for finding the solution to the subtraction problem should be recorded on the chalkboard. The recording should be as follows:

3 tens 13 ones
~~4 tens 3 ones~~

- 1 ten 8 ones

2 tens 5 ones

Give the students other problems to solve, being careful to discuss the solution of each and record the work on the board. During the discussion of each problem it is important that the student recognize that a ten had to be regrouped before the subtraction could be executed. Once the regrouping concept is recognized, then it should be established that this concept is no different than rewriting the number a different way. Make sure the students draw a line completely through the number they are renaming, with the new name written above the number.

**Activity 2:** Give each student bundles of ten sticks. Write a problem similar to 4 tens 0 ones and ask the students to subtract 6 sticks from these sticks. When the students have found the answer have one of them explain to the class how he arrived at his answer. Record the results on the chalkboard.

e.g. 3 tens 10 ones
~~4 tens 0 ones~~
- 6
3 tens 4 ones

Repeat with
other bundles
of ten sticks.

+ + + + + + + + + + + + + + +

**Concept:** Subtraction of whole numbers

**Stage of development:** Abstract

**Equipment:** Worksheet #1

**Activity:** Complete the worksheet.

Name ____________________

1. $\begin{array}{rrrr} & \text{5 tens} & \text{2 ones} \\ - & \text{2 tens} & \text{4 ones} \\ \hline & \text{tens} & \text{ones} \end{array}$

2. $\begin{array}{rrrr} & \text{6 tens} & \text{8 ones} \\ - & \text{3 tens} & \text{9 ones} \\ \hline & \text{tens} & \text{ones} \end{array}$

3. $\begin{array}{rrrr} & \text{9 tens} & \text{4 ones} \\ - & \text{8 tens} & \text{8 ones} \\ \hline & \text{tens} & \text{ones} \end{array}$

4. $\begin{array}{r} 46 \\ -28 \\ \hline \end{array}$

5. $\begin{array}{r} 52 \\ -39 \\ \hline \end{array}$

6. $\begin{array}{r} 71 \\ -63 \\ \hline \end{array}$

7. $\begin{array}{r} 85 \\ -\ 7 \\ \hline \end{array}$

8. $\begin{array}{r} 91 \\ -37 \\ \hline \end{array}$

9. 50 = 4 tens ____ ones

10. 63 = ____ tens 13 ones

11. 134 = 1 hundred 2 tens ____ ones

12. ____ = 3 tens 10 ones

13. 130 = 1 hundred 2 tens ____ ones

14. 240 = 2 hundreds 3 tens ____ ones

15. ____ = 4 tens 10 ones

16. ____ = 4 tens 20 ones

17. $\begin{array}{rrrr} & \text{7 tens} & \text{0 ones} \\ - & & \text{8 ones} \\ \hline & \text{tens} & \text{ones} \end{array}$

18. $\begin{array}{rrrr} & \text{4 tens} & \text{0 ones} \\ - & \text{1 ten} & \text{7 ones} \\ \hline & \text{tens} & \text{ones} \end{array}$

19. $\begin{array}{rrrr} & \text{8 tens} & \text{0 ones} \\ - & \text{7 tens} & \text{8 ones} \\ \hline & \text{tens} & \text{ones} \end{array}$

20. $\begin{array}{r} 40 \\ -16 \\ \hline \end{array}$

21. $\begin{array}{r} 75 \\ -38 \\ \hline \end{array}$

22. $\begin{array}{r} 90 \\ -69 \\ \hline \end{array}$

23. $\begin{array}{rrrr} & \text{3 hundreds} & \text{6 tens} & \text{2 ones} \\ - & & \text{4 tens} & \text{8 ones} \\ \hline & \text{hundreds} & \text{tens} & \text{ones} \end{array}$

Write 2 hundreds 4 tens 0 ones in different ways on the back of the paper.

**Concept:** Subtraction of whole numbers

**Stage of development:** Abstract

**Equipment:** Worksheet #2

**Activity:** Complete the worksheet.

Name ________________

1. 7 tens 2 ones
−2 tens 8 ones
tens ones

2. 7 tens 0 ones
−3 tens 6 ones
tens ones

3. 5 tens 0 ones
−1 ten 3 ones
tens ones

4. 4 7
−2 8

5. 8 0
−6 4

6. 9 3
−7 6

7. 6 6
−5 9

8. 9 0
−3 9

9. _____ = 12 ones 5 tens

10. _____ = 6 tens 23 ones

11. 432 = 3 hundreds _____ tens 2 ones

12. 647 = 6 hundreds 3 tens ____ ones

13. 700 = 6 hundreds _____ tens 0 ones

14. _____ = 2 hundreds 10 tens 3 ones

15. _____ = 2 hundreds 8 tens 10 ones

16. 2 hundreds 6 tens 4 ones
− 3 tens 7 ones
hundreds tens ones

17. 4 hundreds 6 tens 8 ones
− 7 tens 3 ones
hundreds tens ones

18. 6 hundreds 2 tens 7 ones
− 9 tens 3 ones

19. 4 hundreds 0 tens 5 ones
− 6 tens 2 ones

20. 3 2 4
− 3 1

21. 5 4 8
− 2 9

22. 1 3 7
− 8 6

23. 2 4 5
− 1 8

**Concept:** Subtraction of whole numbers

**Stage of development:** Abstract

**Equipment:** Worksheet #3

**Activity:** Complete the worksheet.

Name ____________________

1. 2 hundreds 4 tens 8 ones
   − 6 tens 4 ones

2. 3 hundreds 5 tens 6 ones
   − 8 tens 2 ones

3. 4 hundreds 7 tens 3 ones
   − 5 tens 7 ones

4. 3 hundreds 2 tens 4 ones
   − 7 tens 7 ones

5. 232 = 2 hundreds 2 tens ____ ones

6. 321 = 2 hundreds ____ tens 1 one

7. _____ = 1 hundred 13 tens 6 ones

8. _____ = 1 hundred 2 tens 16 ones

9. 300 = 2 hundreds ____ tens 0 ones

10. 240 = 2 hundreds 3 tens ____ ones

11. _____ = 2 hundreds 9 tens 10 ones

12. Write 2 hundreds 0 tens 0 ones in five different ways:
    1. _____ hundreds _____ tens _____ ones
    2. _____ hundreds _____ tens _____ ones
    3.
    4.
    5.

13. 2 hundreds 0 tens 4 ones
    − 7 tens 2 ones

14. 3 hundreds 0 tens 7 ones
    − 2 tens 5 ones

15. $\begin{array}{r} 2\ 6\ 5 \\ -\ \ 8\ 4 \\ \hline \end{array}$

16. $\begin{array}{r} 3\ 4\ 1 \\ -\ \ 7\ 2 \\ \hline \end{array}$

17. $\begin{array}{r} 4\ 2\ 3 \\ -1\ 6\ 2 \\ \hline \end{array}$

18. $\begin{array}{r} 3\ 0\ 4 \\ -\ \ 7\ 1 \\ \hline \end{array}$

19. $\begin{array}{r} 2\ 8\ 0 \\ -\ \ \ \ 7 \\ \hline \end{array}$

20. 2 8 2 − 3 9 = _____

## CLASSROOM ACTIVITIES

**Concept:** Subtraction of two-digit numbers (no regrouping)

**Stage of Development:** Concrete

**Equipment:** Sticks, rubber bands, boxes labeled tens and boxes labeled ones

**Activity:** The teacher prepares some bundles of ten sticks and leaves the rest as single sticks. He then writes on the board a subtraction problem involving two-digit numbers (e.g., $34 - 23 = \square$). A child can then represent the minuend (34), using sticks. He should use three bundles of ten sticks and four single sticks. He then performs the act of subtraction by taking away the number of sticks of the subtrahend (23). This means he will take away two of the three bundles of ten sticks and three of the four single sticks, leaving one bundle of ten and one single stick. Therefore, the answer is 11. It is important that the children understand the process of subtraction of two-digit numbers where no regrouping is necessary, before attempting problems involving regrouping.

* * *

**Stage of Development:** Concrete

**Equipment:** Place-value device (no regrouping)

**Activity:** The teacher writes a subtraction problem on the chalkboard. It will show two-digit numbers that do not involve regrouping (e.g., $45 - 21 = \square$). One child should represent the minuend (45) on the place-value device, using beads. He will need to use four rods of ten and five singles. Someone else can take away the number of beads shown in the subtrahend (21). He should take two rods of ten and one single. This leaves two rods of ten and four singles on the place value device. Therefore, the answer is 24.

* * *

**Concept:** Subtraction of two-digit numbers (with regrouping)

**Stage of Development:** Concrete

**Equipment:** Sticks

**Activity:** The children are given a subtraction problem involving two-digit numbers, where regrouping is involved in finding the answer (e.g., 63 - 27 = $\boxed{\phantom{00}}$). After the child has represented 63 with sticks (six bundles of ten and three singles), he finds that he cannot take seven sticks away from three sticks. The children will then need time to determine how this difficulty can be overcome. Hopefully, someone will suggest taking the rubber band off of one of the bundles of ten sticks to make more single sticks (in order that the subtraction may be accomplished). After taking off the rubber band, the child has five bundles of ten and thirteen singles - from which he takes away two tens and seven singles, leaving three tens and six singles. The answer is 36.

********************************

**Stage of Development:** Concrete

**Equipment:** Place-value device

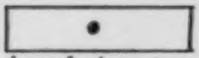
**Activity:** The teacher writes a subtraction problem on the chalkboard, using two-digit numbers that will involve regrouping in the process of getting the answer (e.g., 63 - 26 = $\boxed{\phantom{00}}$). The child represents the minuend by using six rods of ten beads and three single beads. He will run into difficulty when he attempts to take away the six singles of the subtrahend from the three singles of the minuend. Hopefully, someone will suggest converting one of the rods of ten into ten singles, giving five rods of ten and thirteen singles for the minuend. The child can now take away the subtrahend, leaving three rods of ten and seven singles. Thus, the answer is 37.

********************************

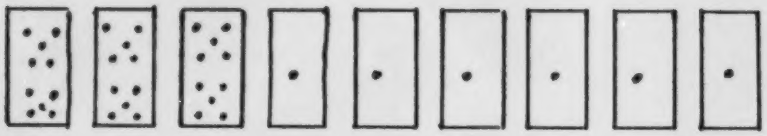
**Concept:** Subtraction of two-digit numbers (no regrouping)

**Stage of Development:** Semiconcrete

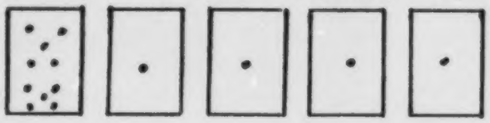
**Equipment:** Individual flannel boards, sets of ten and sets of one

**Activity:** Rectangular strips of construction paper showing a set of ten dots and a set of one dot  are prepared. A subtraction problem involving two-digit numbers and no regrouping (e.g., 36 - 22 = $\boxed{\phantom{00}}$) is given to the children.

They are to find the answer, using the paper strips on their flannel boards. They should represent the minuend (36) as shown:


.

After taking away the subtrahend (22), they should have left


.

Therefore, the answer is 14.

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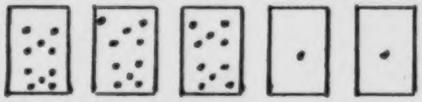
**Concept:** Subtraction of two-digit numbers (with regrouping)

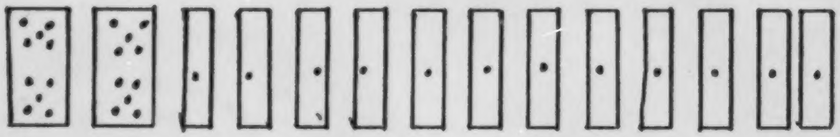
**Stage of Development:** Semiabstract

**Equipment:** Individual flannel boards, rectangular strips showing sets of ten and sets of one

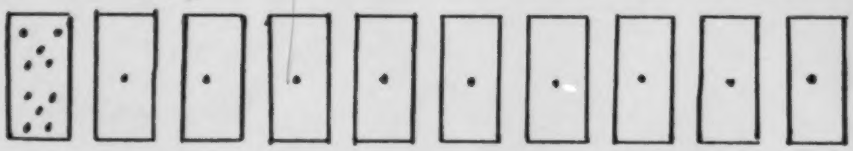
**Activity:** A subtraction problem involving two-digit numbers and regrouping (e.g., 32 - 13 = ▭ ) is given to the children. They are then asked to find the answer, using the paper strips on their flannel boards. The minuend would be represented as


.

The children will run into difficulty when they want to take away the three ones of the subtrahend from the two ones of the minuend. The children should help the teacher decide how this can be done. Someone will suggest exchanging one of the sets of ten for sets of one. The minuend now looks like this:


.

The children can now take away the subtrahend, leaving


.

Therefore, the answer is 19.

********************************

| | |
|---|---|
| Stage of Development: | Semiabstract |
| Equipment: | Individual chalkboards |
| Activity: | The teacher writes a subtraction problem, involving two-digit numbers, on the chalkboard. The children then copy the problem on their chalkboards and find its answer. When they are finished they should hold their chalkboards in the air for the teacher's observation. |

********************************

Refer to activities and materials used in regrouping for addition and adapt to subtraction.

# EQUIVALENT SETS AND SUBSETS

## Introduction

If the elements of two sets can be matched in pairs so that no elements in either set are unpaired, then the sets are in one-to-one correspondence or equivalent. If this correspondence exists between several sets, they are all equivalent. A part of a set is called a subset. Equivalent subsets are obtained when sets with the same number of elements are formed (using elements from a given set). The study of equivalent subsets gives the basis of the concept of multiplication, which is the number operation associated with the joining of these equivalent subsets. If, for example, five sets of two elements are joined, the five names the number of sets, the two is the number in each set, and ten is the number of the union of the equivalent sets. The related number operation can be written as $2 + 2 + 2 + 2 + 2 = 10$. This section will develop the basic concept of multiplication.

**Concept:** Equivalent subsets

**Stage of Development:** Concrete

**Equipment:** Blocks - construction paper circles

**Activity:** Children are given blocks and paper circles and are asked to make equivalent sets using as many counters and sets as they wish.(Example - Three sets of four)

+ + + + + + + + + + + + + + +

**Concept:** Equivalent subsets

**Stage of Development:** Concrete

**Equipment:** Quiet counters, paper plates

**Activity:** Children are given designated number of plates and quiet counters. Using the sets (example - 3, 4, 5, 6, etc.), students make equivalent sets of counters on their plates and verbalize the results mathematically. "I have four sets of five." "I have three sets of two."

+ + + + + + + + + + + + + + +

**Concept:** Equivalent sets

**Stage of Development:** Concrete

**Equipment:** Rings and teddy bears

**Activity:** Children place teddy bears in construction paper rings and verbalize the number of sets, number of counters and the number all together. Example - three sets of six makes eighteen. Children continue the above activities until all facts have been discovered. Then teacher would direct the activity by saying " how me three sets of four". "How many sets do you have?" "How many counters?" "How many do you have all together?"

+ + + + + + + + + + + + + + +

**Stage of Development:** Concrete

**Equipment:** Egg cartons segmented into different numbers of compartments, small counters (beads, discs, marbles, etc.)

Activity: Each child is given a portion of an egg carton (e.g., three). After doing this, a child can make a verbal statement concerning the number of compartments, the number of counters in each compartment, and the number of beads in all the compartments. ("Five compartments of three beads makes fifteen beads.") At a later stage, the children may describe the above example by using the terms, "times" and "equals" (five times three equals fifteen). The multiplication facts for five can be learned by having the children place a different number of counters in each of five compartments. The multiplication facts for any other number can also be learned by using that number of compartments and using a different number of elements in each compartment.

+ + + + + + + + + + + + + + + +

Stage of Development: Concrete

Equipment: Geoboards, rubber bands

Activity: Each child is given a geoboard and several rubber bands. The children then form three equivalent sets (sets having the same number of elements) on their geoboards, using the rubber bands to form the sets. It is important that the child does not use the same nail in more than one set. One child can then be asked to tell the number of sets formed, the number of nails in each set, and the total number of nails enclosed by the rubber bands. If the child enclosed four nails in each of the three sets, he would then say, "Three sets of four nails makes twelve nails."

+ + + + + + + + + + + + + + + +

Concept: Equivalent subsets

Stage of Development: Concrete

Equipment: Jacks

Activity: Children use the jacks while pretending they are on their threes, fours, etc. They then arrange the jacks in the appropriate sets, verbalizing the number of jacks, number of sets and how many jacks in all.

+ + + + + + + + + + + + + + + +

**Concept:** Equivalent sets

**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, flannel objects, pieces of yarn

**Activity:** Twelve objects are placed near the top of the flannel board. Three enclosed curves are then made, using the pieces of yarn. One child is asked to come forward. He fills these sets with the objects from the top of the flannel board, so that each set contains the same number of elements. Each set should contain four members. The children are now asked to verbalize the number of elements in each set, the number of sets, and the total number of objects. They should say, "Three sets of four makes twelve." This can be repeated by having the children fill two, four, and six sets.

+ + + + + + + + + + + + + + + +

**Stage of Development:** Semiconcrete

**Equipment:** Individual flannel boards, objects for each child

**Activity:** The child is told to place a certain number of flannel objects on his flannel board (e.g., eight). He is then asked to divide these objects into equivalent sets (for the above example, either four or two sets). Another member of the class may give a verbal description of how many sets the first child has, how many objects are in each set, and the total number of objects on the board.

+ + + + + + + + + + + + + + + +

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic shapes

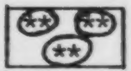
**Activity:** A set of shapes is placed on the magnetic board (e.g., ten). The children are told to arrange these shapes into a certain number of equivalent sets (for the above example either five or two). Then they are asked to give a verbal statement as to the number of equivalent sets, the number of elements in each set, and the total number of pieces on the board. If a child formed two equivalent sets for the above example, he would say, "Two sets of five makes ten."

+ + + + + + + + + + + + + + + +

Concept: Equivalent subsets

Stage of Development: Semiconcrete

Equipment: Magnetic board, cards with sets, cards with number sentences



Activity: Teacher exposes a card to the class and the children will verbalize the number of sets, number of objects in each set and the total number of objects. Teacher asks if someone knows a shorter, easier way to write down what they have verbalized. As children verbalize (e.g. three sets of two), teacher records symbol "x" and "=". A child then finds the card which shows 3 x 2 = 6 and places it on the magnetic board. Use cards and pictures of sets to represent different facts appropriate for the learning level of the children.

+ + + + + + + + + + + + + + + +

Concept: Equivalent sets

Stage of Development: Semiabstract

Equipment: Overhead projector, objects to use on the overhead, overhead projector pen

Activity: A set of objects is placed on the overhead projector (e.g., six). A certain number of closed curves is then drawn (for the above example either two or three). One student fills the closed curves with elements from the given set, so that each will have the same number of objects. (If two closed curves are drawn for the above example, then each set would contain three objects.) The child is then asked to give a verbal description of the number of equivalent sets, the number of objects in each set, and the total number of objects on the overhead projector. For the above example the child would say, "Two sets of three makes six."

+ + + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: The children are told to draw four circles on their chalkboards. Two triangles are to be drawn in each circle. One child is asked to tell the number of circles, the number in each circle, and the total number of triangles he has drawn. He should say, "Four circles with two triangles makes eight triangles."

+ + + + + + + + + + + + + + + +

Stage of Development: (Varies with equipment)

Equipment: Overhead projector, flannel board, magnetic board, chalkboard, objects for the boards, or some other method for forming equivalent sets

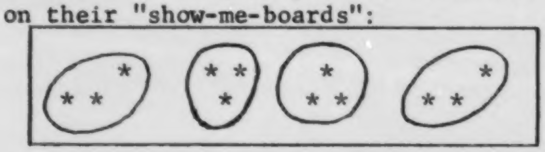
Activity: Several equivalent sets should be formed on one of the above teaching aids (e.g., five sets of three objects). The children are asked to give a verbal description of the joining of these equivalent sets, using the terms "times" and "equals". For the above example, the children should reply, "five times three equals fifteen". They can be given different problems by changing the number of members in the equivalent sets.

+ + + + + + + + + + + + + + +

Stage of Development: Semiabstract

Equipment: Flannel board, magnetic board or chalkboard, objects for the flannel board and magnetic board, "show-me-boards"

Activity #1: Using one of the above boards the teacher will show equivalent subsets to the class. The children are asked to express the joining of these sets as a multiplication problem, using the symbol (x) for multiplication. (e.g., if the following sets are shown to the children, they should form the problem 4 x 3 on their "show-me-boards":



.)

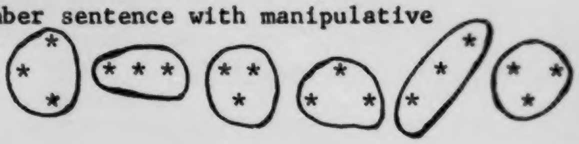
Activity #2: After the equivalent sets are on the chalkboard, the children are required to write a multiplication sentence on their chalkboards. It must show the joining of the sets (3 x 4 = 12 for the preceding example).

+ + + + + + + + + + + + + + +

Equipment: Individual flannel board or individual chalkboard, objects for the flannel board

Activity: A multiplication problem is written on the chalkboard (e.g., 3 x 4 = ?). The children are to form equivalent sets on their flannel boards or chalkboards that will represent this multiplication problem. For the above example, the child should form four sets of three objects. The children must determine the answer to the problem from this representation.

+ + + + + + + + + + + + + + + +

**NOTE:** Whenever possible the teacher should relate the multiplication number sentence with manipulative materials, (e.g.  6 x 3 = 18.)

+ + + + + + + + + + + + + + + +

**Concept:** Equivalent Subsets

**Stage of Development:** Concrete

**Equipment:** M & M's or counters of some kind, containers

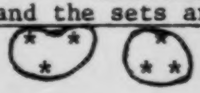
**Activity:** Each child receives counters and a cup. The teacher then asks if the students can fix their counters so that each child has the same number of counters in his cup. Then ask the question, "How many cups do you have?" "How many counters are there in each cup?" "How can you describe the sets?" Children may answer with "There are the same number in each cup" "They are equal." Teacher then asks if there is still another way to describe the sets and leads children to the terms "Equivalent Subsets". Children should then practice with different concrete materials making sets that are equivalent.

+ + + + + + + + + + + + + + + +

**NOTE:** Adopt the activities of this section for the different levels found on the skill sheet. Vary the counters to keep the students' interest.

# MULTIPLICATION FACTS

## Introduction

The meaning of multiplication should be developed by reference, first to the joining of equivalent sets (e.g., when four different sets of six are joined, the cardinal number of the union is 24), then to the addition of equal addends. (e.g., a short method for naming the sum 6 + 6 + 6 + 6 is to use the multiplication fact 4 x 6 = 24). Before working with numbers in multiplication problems, the child should be given many experiences in joining equivalent sets. Then he can be shown how multiplication problems are related to these experiences. (e.g., 2 x 3 means two equivalent sets of three elements.) It is sometimes easier for the child to join the equivalent sets representing a multiplication problem if the sets are arranged in an array. An array is where the elements of the sets are placed in a row and the sets are positioned under each other. (e.g., the equivalent sets  would look like * * * in an array).
* * *

The array above is described as a 2 by 3 array. Some activities involving arrays will be given in this section.

A third method of describing multiplication may be established from the general concept that every number has many names (conservation of numbers). The number 24 can be given several different multiplication names: 1 x 24, 2 x 12, 3 x 8, 4 x 6, etc. In general, then, multiplication may be described as the renaming of two factors as a product. One of the factors corresponds to the number of equivalent sets and the other factor to the cardinal number of each set; the product corresponds to the cardinal number of the union of the sets.

All multiplications of whole numbers may be developed from the basic multiplication facts. Such facts involve two factors less than ten. There are 100 possible combinations, but the number of these which must be learned separately can be reduced by the use of three principles: The product of any number and zero is zero; the product of one and any other number is that number; the order in which two numbers are multiplied does not change the product (2 x 3 = 6, 3 x 2 = 6). The first two principles reduce (by 36) the number of basic facts that need to be memorized. Eight combinations involve a double factor (2 x 2, 3 x 3, etc.). When these have been learned, there are only 56 facts left. Because the order does not affect the product, only half of these 56 facts need to be learned separately. Thus, there are finally only 36 basic combinations to memorize. This section contains activities that allow the child many opportunities to learn these facts.

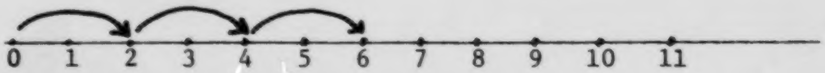
## MULTIPLICATION - USING ARRAYS

In the preceding pages, the activities involved the forming of arrays for a given set or number, and then stating the multiplication number sentence associated with the array. The process can now be reversed by giving the child a multiplication problem and requiring him to form an array for it (e.g., give him 3 x 4 and he forms the array .... .... .... ). This will enable the child to find the answer to the multiplication problem by finding the number of elements in the array.

+ + + + + + + + + + + + + + + +

## MULTIPLICATION - NUMBER LINE

Multiplication problems can be stated as an equal addends problem. Therefore, the product can be reached by finding the sum of these addends. The number line can be used to find the sum of the addends, as described in the section on number lines. The number line gives the child another method of obtaining products. (e.g., 3 x 2 = _____, is the same as 2 + 2 + 2 = _____ whose solution on the number line is shown below:



2 + 2 + 2 =6 ⟷ 3 x 2 = 6

+ + + + + + + + + + + + + + + +

## MULTIPLICATION - COUNTING

Most children entering the intermediate grades already know how to count by twos and by fives. Practice in counting by threes, fours, sixes, sevens, eights, and nines, can be helpful in teaching the possible products for any specific multiplier. (Based on equal addends.)

+ + + + + + + + + + + + + + + +

## UNDERSTANDING THAT MULTIPLICATION IS COMMUTATIVE

### Introduction

Knowing and understanding the concept of the commutativity of multiplication will require the child to learn only half as many multiplication facts. This is true because every non-double multiplication fact is one of a pair involving the same two numbers (e.g., 3 x 5 = 15 and 5 x 3 = 15 or 6 x 4 = 24 and 4 x 6 = 24, etc.). Therefore, when a child knows one fact, he automatically knows two facts.

+ + + + + + + + + + + + + + + +

**Concept:** Multiplication - using equivalent sets

**Stage of Development:** Concrete

**Equipment:** Quiet counters

**Activity:** The basic concept of multiplication is developed in the preceding section on equivalent sets. All work with multiplication should be preceded with the study of this section. After the child has an understanding of the relationship of multiplication and equivalent sets, the use of arrays can be introduced to help him find the multiplication products. Extensive activity with equivalent sets should go on at this time.

+ + + + + + + + + + + + + + +

**Concept:** Multiplication - using arrays

**Stage of Development:** Concrete

**Equipment:** Counters (blocks, discs, buttons, etc.)

**Activity:** Each child is given the same number of counters (e.g., 12). He is asked to form an array with his counters. Members of the class can then be called on to show their arrays and to write the corresponding multiplication sentences on the chalkboard. Once the first sentence has been written, the teacher should ask if anyone else has an array that represents the same sentence. If so, he is asked to show his array. A student with a different array then comes to the chalkboard and writes his sentence. This process is repeated until all students have shown their arrays. (The possible arrays for twelve counters are: 1 x 12, 2 x 6, 3 x 4, 4 x 3, 6 x 2, and 12 x 1). The students must count the members of an array by multiplying the number of rows times the number of objects per row. (e.g., three rows of four would be represented by 3 x 4).

+ + + + + + + + + + + + + + +

**Concept:** Multiplication - using arrays

**Stage of Development:** Concrete

**Equipment:** Geoboards, rubber bands

**Activity:** A number is written on the chalkboard (e.g., six). The children are told to use a rubber band to enclose an array of nails. For the above example, six nails should be enclosed. They then hold up their geoboards for the teacher's observation. Those who have correct arrays may tell the multiplication number sentences associated with their arrays. All possible arrays should be found.

+ + + + + + + + + + + + + + + +

Concept: Multiplication - using arrays

Stage of Development: Concrete

Equipment: Demonstration pegboard, pegs

Activity: A number is written on the chalkboard (e.g., 18). The children use pegs to form an array having 18 pegs. Some possible answers for this array are: 2 x 9, 3 x 6, 18 x 1, etc. After each array is formed, the children should be asked to state the multiplication sentence associated with the array.

+ + + + + + + + + + + + + + + +

Concept: Multiplication - using arrays

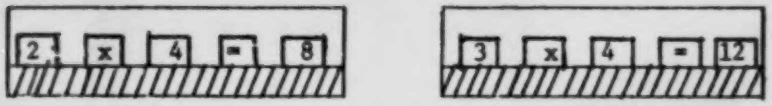
Stage of Development: Semiconcrete

Equipment: Magnetic board, ping pong balls with magnetic strips, show-me-boards

Activity: Place a certain number of ping pong balls on the magnetic board. Have someone form an array with the balls. The other members of the class are required to form a multiplication sentence for the array on their show-me-boards. (e.g. for the array:

```
      * * * *          * * * *
      * * * *          * * * *
                       * * * *   )
```

The corresponding number sentences are 2 x 4 =8, 3 x 4 = 12.



Show-me-boards

+ + + + + + + + + + + + + + + +

**Concept:** Multiplication - using arrays

**Stage of Development:** Semiconcrete

**Equipment:** Individual flannel boards, flannel objects

**Activity:** Each pupil is given the same number of flannel objects. The teacher verbalizes a multiplication sentence while the pupils form a corresponding array of flannel objects for the number sentence on their flannel boards. This process is continued until all combinations are found for the number of given objects. (2 x 3 = 6; 3 x 2 = 6; 1 x 6 =6; 6 x 1 = 6; if six objects are given to the student).

+ + + + + + + + + + + + + + +

**Concept:** Multiplication - using arrays

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, objects (pennies, pegs, counters, etc.)

**Activity:** A certain amount of objects are placed on the overhead projector (e.g., eight). A child is asked to form an array with these objects. Another child can give the multiplication number sentence associated with this array. This should be continued until all the combinations of two factors of eight are found.

+ + + + + + + + + + + + + + +

**Concept:** Multiplication - using arrays

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** The children are told to write a certain number at the top of their chalkboards (e.g., eight). Then they form an array having this number of dots (circles, triangles, etc.) below the number. The multiplication sentence associated with the array should be written at the bottom of the chalkboards. The children then hold their chalkboards in the air for the teacher's observation. All of the different solutions are recognized.

+ + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Abstract

Equipment: None

Activity: Put five of the students' names on the chalkboard in the following manner:

| | May | Sue | Dan | Kay | Jan |
|---|---|---|---|---|---|
| green crayon | ● | ● | ● | ● | ● |
| yellow crayon | ● | ● | ● | ● | ● |
| blue crayon | ● | ● | ● | ● | ● |

With the aid of the chart, students will answer the following questions asked by the teacher.

1. How many green (yellow, blue) crayons are there? (Five, five, five, five, five)
2. How many crayons does each student have? (three)
3. How many crayons are there? (5 x 3, 15, 3 x 5)

+ + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Abstract

Equipment: Worksheets

Activity: Have worksheets of charts made up like the example for five shown below:

| Numbers | | | Result |
|---|---|---|---|
| 5 | 0 | | 0 |
| 5 | 1 | | 5 |
| 5 | 2 | | 10 |
| 5 | 3 | | 15 |
| 5 | 4 | | 20 |
| 5 | 5 | | 25 |
| 5 | 6 | | 30 |
| 5 | 7 | | 35 |
| 5 | 8 | | 40 |
| 5 | 9 | | 45 |

| Numbers | | | Result |
|---|---|---|---|
| 0 | 5 | | 0 |
| 1 | 5 | | 5 |
| 2 | 5 | | 10 |
| 3 | 5 | | 15 |
| 4 | 5 | | 20 |
| 5 | 5 | | 25 |
| 6 | 5 | | 30 |
| 7 | 5 | | 35 |
| 8 | 5 | | 40 |
| 9 | 5 | | 45 |

Be sure to hold the first number constant and then hold the second number constant. Have worksheets available so students can work with charts one through nine.

+ + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Concrete

Equipment: Checkers, checkerboard

Activity: Checkers are pasted to a checkerboard (e.g., three rows of four checkers). The board is held up in front of the students so they can see that there are twelve elements arranged in three rows of four (three times four is twelve). The checkerboard is then rotated in view of the class, so that the rows become columns. The children should be able to tell the multiplication number sentence for the board in its new position (four times three is twelve).

+ + + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Concrete

Equipment: Egg cartons

Activity: Balls of paper are glued into the slots of the egg cartons. An egg carton with balls of paper in it is shown to the class. The children then give the multiplication number sentence for the array of balls of paper (2 x 6 = 12). The egg carton is held up in a vertical position. The children are then asked to give a number sentence for the carton in its new position (6 x 2 = 12). For a different problem involving a factor of two, the egg carton can be cut so there will be four slots instead of six.

+ + + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Concrete

Equipment: Twelve pop bottle caps, graph paper (1" squares)

Activity #1: Pupils form arrays using the caps on the graph paper. The teacher writes the corresponding multiplication sentence on the chalkboard. Repeat the process until all possible arrays for twelve bottle caps are discovered.

Possible arrays:

```
* * *                          * * * *
* * *      4 x 3 = 12          * * * *      3 x 4 = 12
* * *                          * * * *
* * *
```

The children should pair the arrays that involve the same numbers. These pairings would be examples of the commutative property of multiplication.

+ + + + + + + + + + + + + + + +

Activity #2: Same as Activity #1, except pupils can use different numbers of pop bottle caps, (1 - 12). There are 35 possible number sentences.

+ + + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Concrete

Equipment: Geoboards, rubber bands

Activity: The children form an array of nails on their geoboards, using a rubber band. They are then asked to give the multiplication number sentence associated with this array. The sentence is written on the chalkboard. Next, the children rotate their geoboards one fourth of a turn and write a new multiplication sentence on the chalkboard. Both number sentences will have the same product but the factors will be reversed.

```
(e.g.,  * * *              * *
        * * *   becomes    * *   when rotated, and
                           * *
```

the equations are 2 x 3 = 6 and 3 x 2 = 6). This demonstrates the unimportance of order in multiplication (commutative property).

+ + + + + + + + + + + + + + + +

Concept: Commutative property of multiplication

Stage of Development: Semiabstract

Equipment: Multiplication chart

Activity: The students are shown a multiplication chart that provides all the names for a particular number. (e.g., 1 x 12 = 12; 2 x 6 = 12; 3 x 4 = 12; 4 x 3 = 12; 6 x 2 = 12; 12 x 1 = 12). In each case, the product of the two factors is twelve. It should be emphasized that 3 x 4 = 12 and 4 x 3 = 12. The class may then identify other pairs of numbers having the same product.

**Concept:** Multiplication activities with unifix materials

**Stage of Development:** Concrete

**Equipment:** Pan balance and unifix materials

**Activity #1:** Put three groups of four in one side of a pan balance. One by one put cubes into the other side until it balances. Count as you put them in.

| Size of Group | How many groups | How many cubes |
|---|---|---|
| 3 | 4 | 12 |
| . | . | . |
| . | . | . |
| . | . | . |

**Activity #2:** Take a given number of colors and a given number of cubes of each color. Make a train.

e.g., Take three colors and three cubes of each color.

| R | R | R |   | W | W | W |   | G | G | G |
|---|---|---|---|---|---|---|---|---|---|---|

Make a train.

| R | R | R | W | W | W | G | G | G |
|---|---|---|---|---|---|---|---|---|

How long is your train?

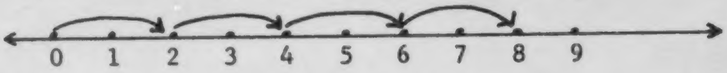
Record Sheet

| No. of Colors | No. of each color | Length of Train |
|---|---|---|
| 3 | 3 | 9 |
| 3 | 2 | 6 |
| 4 | . | . |
| . | . | . |
| . | . | . |

+ + + + + + + + + + + + + + + +

Concept: Number line multiplication

The number line is a valuable pictorial representation, both of the joining of equivalent sets and the addition of equal addends. It also shows the order of the products. This helps the students and also provides some preparation for the concepts of division which will be studied in later grades. Multiplication is a shortcut to addition. Hence, the fact that 2 x 4 is a short way of adding four twos (2 + 2 + 2 + 2) can be shown as follows:



The answer, of course, is eight. It is easily seen from the number line that the product, eight, can also be obtained from 4 x 2. Hence, multiplication is "commutative".

*******************************

Concept: Number line multiplication

Stage of Development: Semiconcrete

Equipment: Magnetic board, a magnetic character

Activity: Draw a number line on the magnetic chalkboard. Give the students a multiplication fact problem to work. Ask one of your students to use the magnetic character to demonstrate the procedure that should be used to find the answer on the number line. Repeat with several other examples.

+ + + + + + + + + + + + + + +

Stage of Develop: Semiabstract

Equipment: Overhead projector

Activity: Have prepared a number line transparencies. Repeat the above procedure. Give the students multiplication problems to solve but ask them to show their number line solution on the overhead projector.

+ + + + + + + + + + + + + + +

Equipment: Individual chalkboards

Activity: Ask the students to draw a number line on their chalkboard. Give them problems to solve on their number lines. They are to show their work to the teacher.

# MULTIPLICATION
# MOTIVATIONAL DRILL RESOURCES

## A. Materials

Counters (all kinds)
Egg cartons
Geo boards
Individual Chalkboards/Flannel boards
Show-me-boards
Peg boards
Multiplication chart
Multiplication flash cards
Magnetic board

## B. Classroom Games (Directions are in the Game Book)

Baseball
Basketball
Concentration
Football
Jeopardy
Tic-Tac-Toe

## C. Interest Center Activities (Directions are in the Game Book)

Airport Math
Battleship
Bowling
Bug-a-roo
Bug-ya
Pokeno
Checker Game
Could Be
Cowboys and Indians
Double Play
Fore
Funny Face
Get It Together
Jack and the Beanstalk
Kung Fu (Lay Around)
Moon Walk
Number Puzzles
Number Toss
O-Rats
Putt-Putt Math
Quiet Dominos
Quizmo
Ring Toss
Rummy
Snag
Snail-o-Math
Space Race
Spill the Beans
Spiral Math
Steal It
Step-a-Set
Toss-a-Cube
Toss It
Twist-o-Math
Uncover
Wichita Win-O
X Marks the Spot

## D. Individual Activities (Directions are in the Game Book)

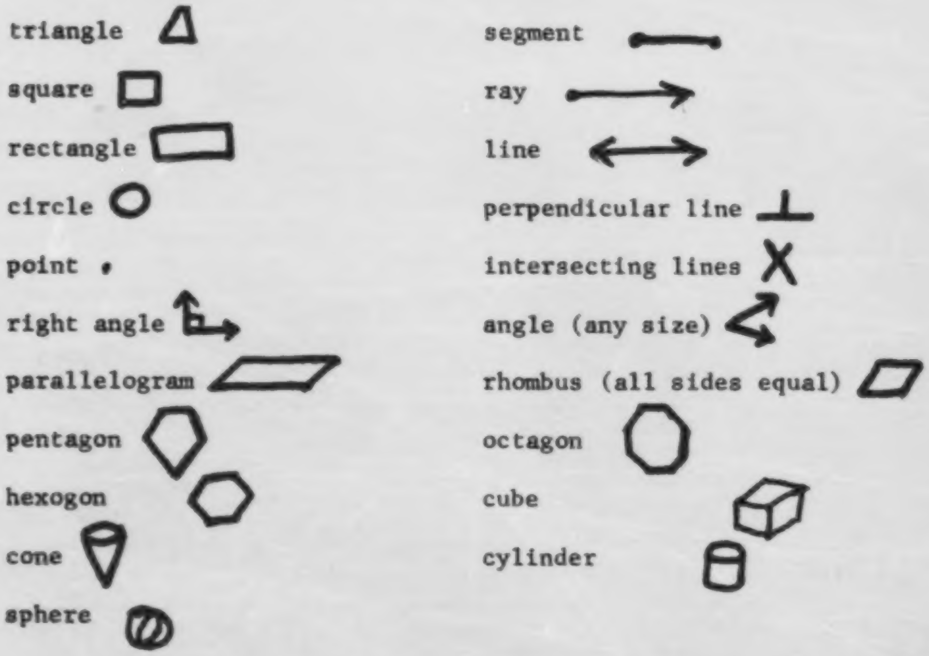
Album Answers
Cardboard Math Machine
Cartridge Math Machine
Clip-a-Pin
Could Be
Dial-a-Number
Fact board
Grid Math
Pocket Math
Rubber Band Number Line
Folder Math
Self-teach Sheet
String-o-Math
Window Math Machine

**NOTE:** Use the devices listed in B, C, and D to reinforce the multiplication facts by using multiplication problems on their playing cards. Provide sufficient practice time on the multiplication facts to insure that the students will know them.

# SYMBOLS

## Introduction

The section of the skill sheet labeled symbols includes those symbols associated with computation, money, measurement and geometry. The learning of the symbols (+), (-), (x) and (÷) is accomplished in addition. subtraction, multiplication and division sections of this book. The symbols ($) and (¢) will be studied in the sections on money. The symbols (%), (#) and (°) can be found in the section on mathematical application. The use of the symbols (<) and (>) will be discussed in this section. The rest of the symbols are geometric. When we are speaking of geometric symbols we are only concerned with the student being able to recognize the geometric figure when drawn on paper or expressed on a two-dimensional surface (flannel board, chalkboard, magnetic board, overhead projector, etc.). Several activities can be found relative to recognizing the shapes: triangle, square, rectangle and circle. The list of geometric symbols are given below.


triangle
square
rectangle
circle
point
right angle
parallelogram
pentagon
hexogon
cone
sphere
segment
ray
line
perpendicular line
intersecting lines
angle (any size)
rhombus (all sides equal)
octagon
cube
cylinder

**Concept:** Symbols < >

**Stage of Development:** Semi-abstract

**Equipment:** Chalkboards

**Activity:** Children are asked to write two numerals on their chalkboard. They are then instructed to make a dot near the smaller numeral. The symbol is then constructed from the dot. (e.g., 4 and 7 are written on the board. A dot is placed by the 4. The symbol is then drawn in using the dot at the point of the symbol,                .) This activity is beneficial in reinforcing the direction the symbol must be made in order to make a true statement.

******************************

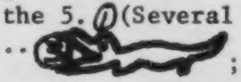
**Concept:** Symbols < >

**Stage of Development:** Semi-abstract

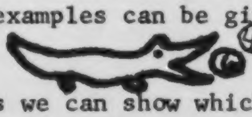
**Equipment:** Chart paper, marking pen

**Activity:** After children have developed an understanding of the concept of "greater than", and "less than", the following story can be developed with the children:

This is Greedy, the alligator. He lives in Math River. He is always very hungry. When Greedy sees numerals floating in the river, he always eats the largest numeral. If he sees a 5 and a 2, he will eat the 5. (Several examples can be given and illustrated...



;



).

When we write what Greedy does we can show which one he eats by drawing only his mouth. When we read what he eats, we are careful to read from left to right. If he eats the first (left hand numeral) we say _____ is greater than ______. When he eats the second numeral (right hand numeral) we say _______ is less than _____. (This story can be further developed using examples of combinations he sees in the river, e.g., 2 + 3, 1 + 2. Greedy would eat 2 + 3, shown 2 + 3 > 1 + 2.)

224

Concept: Greater than, less than

Stage of Development: Semi-concrete

Equipment: Individual flannel boards, objects, flannel pieces showing words: greater than, less than

Activity 1: A set of objects is placed on the demonstration flannel board. The children then are told to form a set on their flannel boards that has "more" objects than the one on the demonstration board. After checking their results, the teacher asks them to form a set that has "fewer" characters. This can be repeated with different sets being placed on the demonstration flannel board. The phrases "greater than" and "less than" can also be used.

Activity 2: Two sets of objects are placed on the demonstration flannel board-leaving a space between the sets. The children put the word card greater than or less than on their flannel boards, making a true comparison of the cardinal numbers of the two sets. Their boards are then held in the air for the teacher's observation.

******************************

Concept: Symbols $>$ , $<$

Stage of Development: Semi-abstract

Equipment: Individual chalkboards

Activity 1: Two sets are placed on the demonstration flannel board. They must be placed next to each other with a space between them. The children write the symbol $>$ , or the symbol $<$ , on their chalkboards (whichever one could be used in the space to compare the cardinal number of each set).

Activity 2: The teacher states two numbers for the children to write on their chalkboards. They then write the symbol $>$ , or the symbol $<$, between the two numbers, to make a true statement.

Activity 3: The children are given one number and either the symbol $>$ or $<$. They then choose a number that will make a true inequality and write it on their chalkboards.

******************************

Stage of Development: Semi-abstract

Equipment: Number lines

Activity: Each child is given a number line. When comparing numbers on the number line, the number which is the closest to zero will be the smallest number, and the one farthest from zero will be the largest number. Three numbers that are on the child's number line are written on the chalkboard. The children are then asked to use their number lines to determine the smallest number and the largest number. This should be repeated with several other triple numbers.

******************************

Stage of Development: Semi-abstract

Equipment: "Show-me-boards", cards showing numerals and the symbols $>$ , $<$

Activity: The symbol $>$, or the symbol $<$, is held up. The students are told to form a statement on their "show-me-boards" using the symbol and two numerals. When they are finished, their boards should be held up for inspection.

******************************

Stage of Development: Semi-abstract

Equipment: Hundreds board, cover for numerals

Activity: The cover is placed on one of the numerals. One child points to a number that is (larger/greater) than the one that is covered. Several numbers that are (less/smaller) than the one that is covered should also be pointed out.

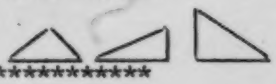
******************************

Concept: Geometry - shapes

Stage of Development: Semi-concrete

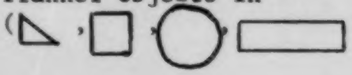
Equipment: Flannel board, flannel cutouts of triangles, squares, rectangles , and circles

Activity: It is necessary to have at least five cutouts of each shape. These shapes are arranged randomly on the flannel board. One pupil finds all the triangles on the board and gives the cardinal number of this set of triangles. All of the squares, rectangles, and circles should also be located. The cardinal number of the set must be stated each time.

Note: All the figures should not be alike. Make the triangles as follows: 

******************************

Stage of Development: Semi-concrete

Equipment: Individual flannel boards, flannel objects in different geometric shapes (  )

Activity: Each child is given a flannel board and a set of geometric shapes. He is told to place a triangle on his flannel board and then to hold the board in the air for the teacher to see. The same instructions are repeated for the other geometric shapes. The activity can be altered by giving the children different-colored shapes. They could then be told to put a red triangle, a green square, or a blue circle on their flannel boards.

******************************

Stage of Development: Semi-concrete

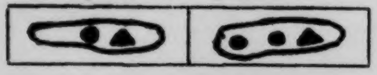
Equipment: Magnetic board, magnetic geometric shapes

Activity: One of the geometric shapes is placed on the magnetic board. Someone is then asked to identify the shape. This is repeated with the other shapes. Then all the shapes are taken off the board. Someone is now asked to put a triangle on the board. This is repeated with all the geometric shapes.

******************************

**Stage of Development:** Semi-concrete

**Equipment:** Set dominos (with sets on each end). There are 36 dominos, which will provide several alternate grouping patterns for each number, zero through six. The elements of the dominos are geometric shapes. The shapes are circle, triangle, and square. A possible pattern follows:



**Activity:** A domino is held up in such a way that only one end is visible to the class. For example, the above domino can be held up with the left end covered, and the teacher can ask the question, "How many circles are visible?" The answer should be two. The teacher can question the class about the other shapes, too.

******************************

**Stage of Development:** Semi-concrete

**Equipment:** None

**Activity:** The children are told to search the room for shapes that are similar to the geometric shapes being studied (circle, square, rectangle, and triangle). A contest can be made out of this by seeing who can find the most shapes.

******************************

**Concept:** Geometry - shapes

**Stage of Development:** Semi-abstract

**Equipment:** Overhead projector, geometric shapes (triangle, square, rectangle, and circle)

**Activity 1:** One of the geometric shapes is placed on the overhead projector. Someone is asked to identify it. A different shape is then put on the projector and the procedure repeated. This is continued until all the different shapes have been identified.

**Concept:** Geometry - shapes

**Stage of Development:** Concrete

**Equipment:** Geo boards, rubber bands

**Activity:** The children are told to form a triangle on their geo boards, using a rubber band. Some of the properties of a triangle should then be discussed. (How many sides? How many corners?). Next, the children form a rectangle on their geo boards. Its properties are discussed. Finally, a square is formed and its properties are discussed.

******************************

**Stage of Development:** Concrete

**Equipment:** Triangular, circular, square, and rectangular shapes cut out of plywood, tactile box

**Activity:** The different shapes are placed inside the box. A child is asked to reach inside the box and, without looking, bring out the triangle. Another student reaches inside and brings out the circle. This procedure is repeated for the other two shapes.

**Activity 2:** A transparency showing several triangles, squares, rectangles, and circles is prepared. One child is given a projector pen and is told to put an "x" on all the triangles. This can be repeated by having someone put a "+" on all the squares. Something similar is done for the rectangles and circles.

******************************

**Stage of Development:** Semi-abstract

**Equipment:** Individual chalkboards

**Activity:** The children are told to draw triangles on their chalkboards. They must be able to tell the number of corners that a triangle has. Then, they draw rectangles. They should hold their chalkboards in the air fcr the teacher's observation. Last, they draw circles and squares. These two will cause them some difficulty.

**Note:** The activities for learning the goemetric symbols were centered on the geometric shapes, triangle, square, circle and rectangle. The same activities can be used for all the different geometric symbols by adapting the activity to fit the symbol.

# CLASSIFICATION OF NUMBERS

## Introduction

All whole numbers can be separated into two sets. The set of composite numbers and the set of prime numbers. Prime numbers are those numbers that are only divisible by themselves and one. All the other whole numbers are called composite for they have factors other than themselves and one, ex. 2, 5, 7, 11 are prime numbers, 6, 15, 25, 48 are composite numbers.

Common fractions are those fractions that have no whole number parts, ex. 3/5, 4/7, 7/4, 18/15, etc.

Decimals are fractions that have denominators which are powers of ten, ex. 3/10, 27/100, 3/100, 127/10, 4365/100, etc., these numbers are usually expressed with a decimal point, .3, .27, .03, 12.7, 43.65, etc.

This section will be concerned with the identification of the different classes of numbers (composite, prime, common fractions or decimals).

| | |
|---|---|
| Concept: | Classification of numbers |
| Stage of Development: | Semiconcrete |
| Equipment: | Magnetic board, magnetic cards showing composite, prime, common fractions or decimals |
| Activity: | Place several of the magnetic cards on the magnetic board. (Place them randomly, not in a row.) Draw four large closed curves on the magnetic board. Label the curves composite, prime, common fractions and decimals. A student is asked to identify all the composite numbers and place them inside the closed curve labeled composite. The same procedure should be used for the prime numbers, common fractions and decimals.<br>Repeat the activity with other numbers. |

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Classification of numbers |
| Stage of Development: | Semiabstract |
| Equipment: | Individual chalkboards |
| Activity: | Write several numbers on the chalkboard. These should include common fractions, composite numbers, prime numbers, and decimals. The students are required to copy the numbers on the board, placing all the numbers that are alike in sets on their chalkboards. The students will show their work to the teacher by holding their chalkboards in the air for observation. |

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Classification of numbers |
| State of Development: | Semiabstract |
| Equipment: | Individual chalkboards |
| Activity: | The students are asked to write a composite number on their chalkboard. The teacher checks the students' work by having the student hold their chalkboards in the air for his/her observation. Repeat with other requests for composite numbers, then ask for prime numbers, common fractions or decimals. |

+ + + + + + + + + + + + + + + +

**Concept:** Classification of numbers

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector

**Activity:** Use the same procedure as the above activity, except have a student write their numbers on the overhead projector for all to see.

+ + + + + + + + + + + + + + +

**Concept:** Classification of numbers

**Stage of Development:** Semiabstract

**Equipment:** 4" x 6" tagboard cards with common fractions, decimals, composite numbers or prime numbers on them.

**Activity:** Place the cards on the chalkboard tray in such a way that they are exposed to the class. Choose two players to be in competition. The two students face the cards that have been placed in the chalkboard tray from a distance of about 10-15'. The teacher makes a statement such as "I am thinking of a number that is called a prime number." The first student to walk to the chalkboard tray and hold up a card showing a prime number is the winner. Two other players can be chosen to be in competition or the winner of the first contest can be paired with another contestant. Repeat until all have had an opportunity to be a participant.

+ + + + + + + + + + + + + + +

**Concept:** Classification of numbers

**State of Development:** Semiabstract

**Equipment:** Game called Concentration

**Activity:** Load the Concentration game board with cards showing fractions, decimals, prime numbers or composite numbers. Play the game by having students pair up two numbers that are prime, composite, fractions, or decimals. In other words pairing up similar numbers.

+ + + + + + + + + + + + + + +

## ROMAN NUMERALS

When working with Roman numerals the student should remember the following:

1. Repeated numerals or symbols are used. (e.g., I, II, III, etc.)
2. The subtractive principle is used. (e.g., IV, 1 from 5 is 4). When the first of two symbols represents a smaller number than the second, you subtract the value of the first from the second to get the value of the combination.
3. The addition principle is used. When the first of two symbols represents a larger number than the second, you add the value of both numbers to get the combination (e.g., VI → 5 + 1 = 6).
4. To name all numbers up to 30, three different symbols are needed, (I, V and X).
5. To name all numbers seven basic symbols are needed.

| | |
|---|---|
| I - 1 | C - 100 |
| V - 5 | D - 500 |
| X - 10 | M - 1,000 |
| L - 50 | |

6. To write large numerals, repeated addition is used:

CCCXXVIII = 300 + 20 + 5 + 3 = 328

7. Two basic sets (5 and 2) are used:

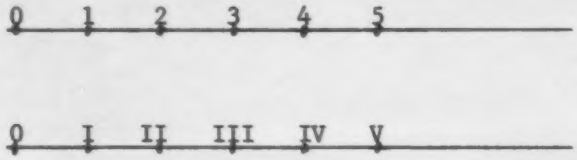
(e.g., 5 = V and 2 times V = X
5 times X = L    2 times L = C
5 times C = D    2 times D = M ).

**Concept:** Roman numerals

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** Each student draws two identical number lines on their chalkboards. They label one with Arabic numerals and the other with Roman numerals. Students then compare the two number lines.



+ + + + + + + + + + + + + + +

**Concept:** Roman numerals

**Stage of Development:** Concrete

**Equipment:** Paper plates, brads, tagboard hands

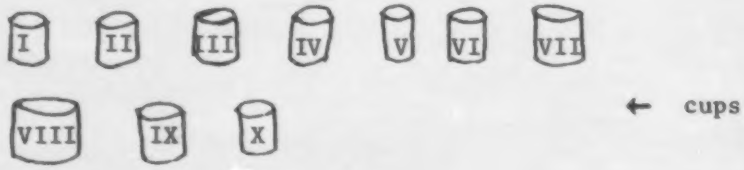
**Activity:** Each student marks off twelve equal divisions on his paper plate. He then labels these points with the Roman numerals (I - XII). After locating the center of the plate he attaches the hour and minute hands. Each student now has a Roman numeral clock. Require the students to set their clocks to read a particular time (e.g., 4:00, 3:30, 6:15, etc.). The students should hold their clocks in the air for your observation.

+ + + + + + + + + + + + + + +

**Concept:** Roman numerals

**Stage of Development:** Concrete

**Equipment:** Counters, ten cups with Roman numerals I to X.



Discuss with your class the various symbols used for Roman numerals. Place a chart showing the seven basic symbols used on the tackboard.

Activity: The students are given a number of counters. They are asked to put as many counters in each cup that corresponds to the Roman numerals. Or, the instructor may choose to have one cup for each student. The student would be asked to place the correct number of objects in the cup that corresponds to its Roman numeral.

+ + + + + + + + + + + + + + + +

Concept: Roman numerals

Stage of Development: Concrete

Equipment: Counting objects

Activity: Give each student a certain number of objects. Require each student to tell how many objects were given to them and represent this number as a Roman numeral. (e.g., three counters, the number is 3, the Roman symbol is III). Repeat the process for other sets.

+ + + + + + + + + + + + + + + +

Concept: Roman numerals

Stage of Development: Concrete

Equipment: Counters, cups or container with ones, tens, hundreds, etc. written on them.

Activity: Students should place a certain number of objects in each container. Explain to the students that no more than nine objects should be in the ones cup. When they have ten objects in the ones cup they will replace them by placing one object in the tens cup. The same procedure would be used when there are ten counters in the tens cup, they would be replaced by one counter in the hundreds cup. Have each student write the Roman numeral that would be used to show how many counters are represented in their containers.

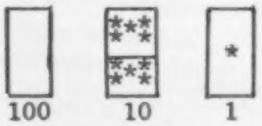
+ + + + + + + + + + + + + + + +

| | |
|---|---|
| **Concept:** | Roman numerals |
| **Stage of Development:** | Semiconcrete |
| **Equipment:** | Individual flannel boards, flannel characters, flannel Roman numerals |
| **Activity:** | The instructor requires each student to place a certain number of objects on their flannel board. (e.g., a set of five objects). They are asked to select the correct Roman numeral that represents the number of characters in the set and place it next to the set on the flannel board. Repeat this process until all the Roman numerals for I to X have been used. |

This can be extended to show thousands, hundreds, tens and ones by using different shapes to represent each of the units. (e.g., □ = 1,000, △ = 100, 0 = 10, ▯ = 1; □ △ 0 ▯ = 1,111 or MCXI).

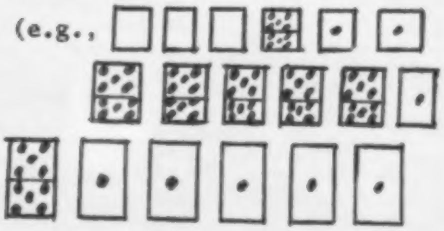
+ + + + + + + + + + + + + + +

| | |
|---|---|
| **Concept:** | Roman numerals |
| **Stage of Development:** | Semiconcrete |
| **Equipment:** | Magnetic boards, magnetic objects, numerals and Roman numerals |
| **Activity:** | The magnetic objects that are to be used could be cards marked in the following manner: |



The teacher places a certain number of hundreds, tens and ones cards on the board while the student places the Arabic and Roman numerals showing how many hundreds, tens and ones the cards represent.

(e.g.,



= 312 = CCCXII

= 51 = LI

= 15 = XV ).

+ + + + + + + + + + + + + + +

**Concept:** Roman numerals

**Stage of Development:** Semiabstract

**Equipment:** Individual flannel boards, overhead projector and overhead projector characters, bundles of counting sticks representing 1's, 10's, 100's, 1000's, individual chalkboards

**Activity:** A certain number of characters are placed on the overhead. The students write the corresponding Roman numeral on their chalkboard. The students will hold their chalkboards in the air for teacher observation of the answers. For larger numbers, sticks are placed on the overhead. (e.g., a bundle of 100, three bundles of ten and four loose sticks for ones). Each student will record on their chalkboard in Roman numeral symbols the number of sticks shown on the overhead.

For the above example, the children would write CXXXIV on their chalkboard. Repeat.

**NOTE!** The concept of place value should be thoroughly understood by the student before using this activity.

+ + + + + + + + + + + + + + +

**Concept:** Roman numerals

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, transparencies

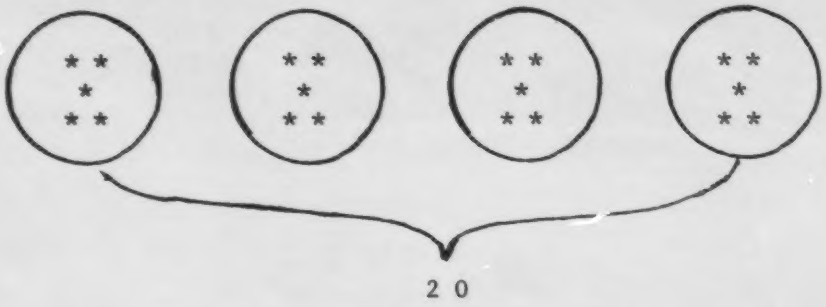
**Activity:** Transparency showing written numerals is placed on the overhead. A student is selected to write the corresponding Roman numeral on the acetate roll. Roman numerals can be written on the acetate roll by the instructor while the student writes the corresponding Arabic numeral. Repeat with other examples.

+ + + + + + + + + + + + + + +

## PARTITIONING SETS

### Introduction

The prerequisite skill for the concept of addition is joining sets, for subtraction it is separating sets and for multiplication it is equivalent sets. To be consistent with these three computational skills, the concept of division should have partitioning sets as a prerequisite skill. An example of partitioning a set would be: You have 20 students in your room and you wish to divide them up into four committees. This could be accomplished by partitioning the 20 students into four sets of five students.



20 partitioned into 4 sets makes 5 in each set

$20 \div 4 = 5$

Concept: Partitioning sets

Stage of Development: Concrete

Equipment: Counters (dice, cubes, sticks, buttons, bottle caps, etc.)

Activity: Give each student two counters. Students should partition their counters into one set. Require the students to verbalize the number of counters in the set. Now have them partition their counters into two equal sets and state the number of counters in each set. Is it possible to partition the counters into more than two sets?

Repeat the above procedure using sets that contain more than two counters. After each attempt at partitioning, the following questions should be answered: How many counters all together? How many different sets? How many in each set? Are there any counters left over?

If you will repeat this procedure with your students for sets of 1 to 81 counters and partition them into one to nine sets, using only the cases when there are no counters left over, your students will be able to discover all of the basic division facts of arithmetic.

Furthermore, if you will construct a chart of the results on the chalkboard, these facts will become apparent. See the sample chart on the following page.

| Total Counters $\boxed{\div}$ | Number of Sets $\boxed{=}$ | Number of Counters in each Set $\boxed{R}$ | Number of Counters left over |
|---|---|---|---|
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| 4 | 1 | 4 | 0 |
| 4 | 2 | 2 | 0 |
| 4 | 3 | 1 | 1 |
| 4 | 4 | 1 | 0 |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| 32 | 8 | 4 | 0 |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |

+ + + + + + + + + + + + + + +

**Concept:** Partitioning sets

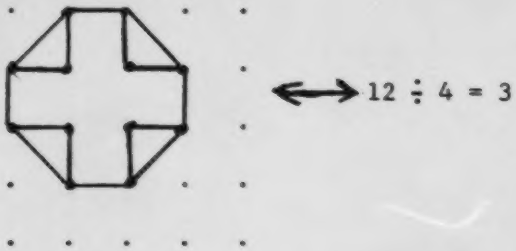
**Stage of Development:** Concrete

**Equipment:** Dried peas or beans, egg cartons

**Activity:** Give each student a certain number of beans and an egg carton. The students then partition these beans into different equal sets, using the compartments of the egg carton to hold the beans. e.g. If you give the student six beans, he will partition the beans into one set, two sets, three sets, four sets, five sets and six sets. When the partitioning leaves no beans left over, you have identified one of the basic division facts. Six (6) beans partitioned into 2 sets of beans leaves no beans remaining would represent the division fact $6 \div 2 = 3$.

+ + + + + + + + + + + + + + +

| | |
|---|---|
| **Concept:** | Partitioning sets |
| **Stage of Development:** | Concrete |
| **Equipment:** | Geoboards, rubber bands |
| **Activity:** | Give each student a geoboard and several rubber bands. Each student encloses a certain number of nails of the geoboard with a rubber band. He is then instructed to partition this set into a number of equal sets using smaller rubber bands. See the illustration below for 12 nails partitioned into 4 sets. |



Using different colored rubber bands for the partitioned sets makes for their easy identification.

Using a 5 x 5 geoboard limits the study of the division facts to 25 ÷ 5 level.

Use butter tubs, paper plates, or other containers to contain the different sets.

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| **Concept:** | Partitioning sets |
| **Stage of Development:** | Semiconcrete |
| **Equipment:** | Magnetic chalkboard, magnetic characters |

Place a certain number of characters on the magnetic board. Require the students to partition this set into a certain number of equal sets. e.g. Given 10 characters partition into 5 sets. Record the total number of characters, the number of sets, how many characters in each set and how many characters are left over after partitioning.

The preceding example would yield the following data: 10 characters partitioned into 5 sets of 2 characters with 0 remaining. Later, this will be identified with the division fact 10 ÷ 5 = 2 or written another way

$$\begin{array}{r} 2 \\ 5\overline{)10} \\ \underline{10} \\ 0 \end{array}$$

Repeat with other sets to establish the basic concept of division.

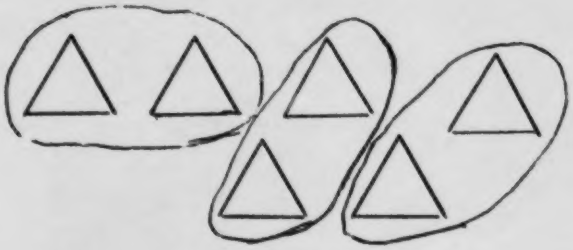
+ + + + + + + + + + + + + + + +

Concept: Partitioning sets

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: Each student is required to draw a certain number of triangles on his chalkboard. Each student partitions his triangles into a certain number of equal sets by drawing an enclosed curve around them.
e.g. Draw 6 triangles on the chalkboard and partition them into three equal sets.



Record your results as: 6 triangles partitioned into 3 sets of 2 triangles which yields, 6 ÷ 3 = 2.

Repeat with other sets to identify other basic facts of division.

+ + + + + + + + + + + + + + + +

Concept: Partitioning sets

Stage of Development: Semiabstract

Equipment: Overhead projector, overhead projector characters

Activity: Place a certain number of characters on the overhead projector. Require a student to partition these characters into different numbers of equal sets. After each act of partitioning, have the student verbalize the number of characters they started with, the number of sets they were partitioned into, the number of characters in each set and the number of characters remaining.

+ + + + + + + + + + + + + + +

# DIVISION FACTS

## Introduction

Multiplication and division are inverse (opposite) operations, for example, 6 x 2 = 12 and 12 ÷ 2 = 6, 8 x 3 = 24 and 24 ÷ 3 = 8. Therefore, the student who knows the basic facts of multiplication and understands the concept of division that is established in the section on partitioning of sets, will have no difficulty learning the basic facts of division. Remember, the basic facts of division are those division problems having divisors and quotients that are a single digit and having no remainder.

```
                                                9 -----quotient
Example:  18 ÷ 2 = 9            or            2 )18
               ¦   ¦                            ¦
         divisor  quotient                   divisor
```

This section will consist of various teaching suggestions for learning the basic computational skills of division. It is not enough that the student understands the basic concept of division, he should also be able to give the answer to any given division fact, instantaneously.
e.g. If someone asks a student, "what is twenty divided by four?" the student should respond "five" immediately without hesitation. The student really does not know his division facts until he is able to do this. A firm command of the basic division facts is critical for the student's mastery of long divisiin problems.

STAY WITH THE ABOVE FACTS UNTIL THEY LEARN THEM.

**Concept:** Division facts

**Stage of Development:** Concrete

**Equipment:** Counters (discs, buttons, beans, sticks, etc.) Containers (paper plates, paper cups, butter tubs, egg cartons, etc.)

**Activity:** Write one of the division facts on the chalkboard.

(e.g. $12 \div 3 = \boxed{\phantom{00}}$ or $3\overline{)12}$) Give each student several counters and containers. Each student uses his counters and containers to represent the division problems. For the above example, the student would place twelve counters into three containers. He would end up with four counters in each container. Four represents the answer of the division fact. Continue the activity, writing different division facts on the chalkboard.

**Note:** To add variety to this activity, change the counters and containers for the lesson presentation on different days. (e.g. use discs and paper plates one day, beans and egg cartons another day, sticks and butter tubs another day, etc.)

+ + + + + + + + + + + + + + +

**Concept:** Division facts

**Stage of Development:** Concrete

**Equipment:** Geoboards

**Activity:** The basic division facts can be represented by use of rectangular arrays.

e.g. $8 \div 4 = \boxed{2} \longleftrightarrow \begin{array}{r} 2 \\ 4\overline{)8} \end{array}$

. .     would be where 8 dots (dividend) can<br>
. .     be represented as 4 rows (divisor)<br>
. .     and 2 columns (quotient)<br>
. .

Another example might be: $10 \div 2 = \boxed{5}$

. . . . .   10 dots as 2 rows<br>
. . . . .   and 5 columns

Give each student a geoboard and a rubber band for each division fact given, the student uses his rubber band to form a representative rectangular array of nails on his geoboard.

Remember, the total number of nails represents the dividend, the number of rows, the divisor, and the number of columns, the quotient. Repeat for the various division facts, always recording the answer.

+ + + + + + + + + + + + + + +

**Concept:** Division facts

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board or flannel board (characters)

**Activity:** Write on the chalkboard, expose or verbalize one of the division facts to the students. Require one of the students to represent the division fact using the magnetic or flannel board and its characters. The student would do this by first selecting a number of characters equal to the dividend of the problem, then partitioning these characters into a number of sets designated by the divisor. The answer to the division problem will be the number of characters in each set. Make sure that students understand this process as it will play an important part when they study long division.

Repeat the above activity with the different division facts, always identifying the answer.

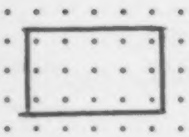
+ + + + + + + + + + + + + + +

**Concept:** Division facts

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector. rectangular coordinate dot grid transparency, ⁝⁝⁝ etc., overhead projector pencil.

**Activity:** Place the dot grid transparency on the overhead projector. Use at least a nine dot by nine dot grid. Write one of the basic division facts on the chalkboard. Someone represents the fact on the transparency by drawing a line around an appropriate rectangular array of dots. e.g. Given the fact $15 \div 3 = \square$ or $3\overline{)15}$, the student would encircle an array similar to the following:



3 rows and
5 columns

Repeat with different basic facts and students.

+ + + + + + + + + + + + + + +

Concept: Division facts

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: Write one of the division facts on the chalkboard. The children are told to draw a rectangular array on their chalkboards that would represent the problem.
e.g. $18 \div 3 = \square \leftrightarrow 3\overline{)18}$, the student would form the array on his chalkboard.

. . . . . . .
. . . . . . .
. . . . . . .

3 rows and 6 columns, with six being the answer to the problem.

Repeat with other problems.

+ + + + + + + + + + + + + + +

Concept: Division facts

Stage of Development: Semiabstract (Individual Activities)

Equipment: Teacher made machines (self teach sheets, cartridge math machine, cardboard math machines, etc.), teaching aids (Clip-O-Pin Factboard, Album Answer, Folder Math, Pocket Math, and String-O-Math), see the Games and Teaching Aids book for construction instructions.

Activity: There are two types of individual activities that students can utilize when learning the basic division facts. They may use materials where they can check their own answers or they may use materials where the teacher has to check the answers for accuracy. Some of the materials that are self checking are cardboard math machines, cartridge math machines and self teach sheets.

On these devices, the division problems are exposed to the students while the answers are hidden from view. After the student has recorded his answer, he checks it against the correct answer.

Some of the devices for individual study that the teacher must check for accuracy are Album Answers, Clip-O-Pin, Factboard, Folder Math, Pocket Math and String-O-Math. These devices all involve the matching of the division fact and its answer. When all the matchings have been made, the teacher must check them for accuracy.

+ + + + + + + + + + + + + + +

Concept: Division facts

Stage of Development: Semiabstract (Small Group Activities)

Equipment: Small group games:

Boing Boing
Car Race
Cat and Mouse
Chalktray Math
Figure 8 Race
Fishing Derby
Flower Game
Funny Face
Game Called War
Grow Doggie
King of the Hill
Lay Around
O Rats
Snail-O-Math
Snoopy
Spill The Beans
Trip To Happy Face
Tug of War

See the Games and Teaching Aids book for construction instructions.

Activity: The small group games listed above can be used to reinforce the learning of basic division facts. Load each of the gameboards with division fact problem cards. The games require a verbal response; therefore, they are well suited for teacher manned learning centers. Be sure and praise students giving correct answers, this positive motivation does wonders for the student's self concept. Students should continue working with the small group games until they have a firm command of the basic division facts. Change learning center games regularly to keep student interest on a high level.

+ + + + + + + + + + + + + + +

Concept: Division facts

Stage of Development: Semiabstract (Large Group Activities)

Equipment:

Baseball Game
Basketball Game
Chalktray Mathematics
Classroom Squares
Football Game
Jeopardy
King of the Hill
Number Concentration
One-0n-One Competition
Tic-Tac-Toe

Activity: The instructions constructing and playing these classroom games are found in the Games and Teaching Aids book. The games are designed to be played with the entire class. Mathematics games should be used as reinforcement tools in the development of mathematical concepts. They are not to be used to initiate the study of new concepts. They are great motivation agents in the study of the basic computational skills.

Load the above gameboards with division fact problem cards. Divide the class into two competitive teams to play the games. Change games regularly to maintain interest. YOU CANNOT GIVE TOO MUCH PRACTICE ON THE BASIC COMPUTATIONAL SKILLS. STAY WITH THEM UNTIL ALL THE STUDENTS HAVE MASTERY.

+ + + + + + + + + + + + + + +

# MULTIPLICATION OF WHOLE NUMBERS BEYOND BASIC FACTS

## Introduction

When the student has a firm understanding of the concepts of place value and regrouping and knows his multiplication facts well, he should have little difficulty working any multiplication problem involving whole numbers.

Use the same classroom games, interest center devices, and teacher-made machines that you used for reinforcement of the multiplication facts for the computation problems involving two digits by one digit, two digits by two digits, three digits by two digits and three digits by three digits numbers. Merely prepare the playing cards for these games to reflect the problems you wish to reinforce.

If a student has difficulties in multiplication of whole numbers beyond basic facts, it is necessary for the teacher to pinpoint the student's problem area. Is it in his multiplication facts or is it where to place his numbers in the answer or is it in column addition?

To determine the problem area, have the student isolate each separate multiplication before he does the entire problem. e.g.,

$$\begin{array}{r} 376 \\ \underline{\times\ 4} \end{array}$$

First step:

$$\begin{array}{r} 300 \\ \underline{\times\ 4} \\ 1200 \end{array} \qquad \begin{array}{r} 70 \\ \underline{\times\ 4} \\ 280 \end{array} \qquad \begin{array}{r} 6 \\ \underline{\times\ 4} \\ 24 \end{array}$$

Second step:

$$\begin{array}{r} 376 \\ \underline{\times\ 4} \\ 1504 \end{array}$$

If the student did the first correctly and not the second step, then the problem is not in multiplication, but in place value.

Note: When using 10 or 100 as a multiplier, the students will soon be aware that the multiplication can be accomplished by moving the decimal point of the multiplicand one place to the right for 10 and two places to the right for 100.

**Concept:** Multiplication of whole number - word problems

**Stage of Development:** Abstract

**Equipment:** None

The teacher should discuss with the student how to pick out the operation that must be performed when they are solving verbal problems. Point out that phrases like "How many in all?" and "How many in 5 boxes?" are often clues to multiplication.

**Activity:** The student can make up verbal problems, using multiplication, for the information contained in charts like the following:

| | | |
|---|---|---|
| Cake | 1 | 4 |
| Cost | 15c | ? |

| | | |
|---|---|---|
| Truck | 1 | 3 |
| Wheels | 14 | ? |

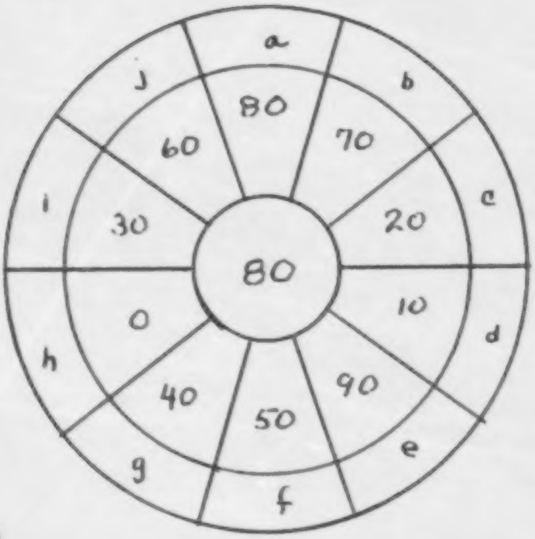
+ + + + + + + + + + + + + + +

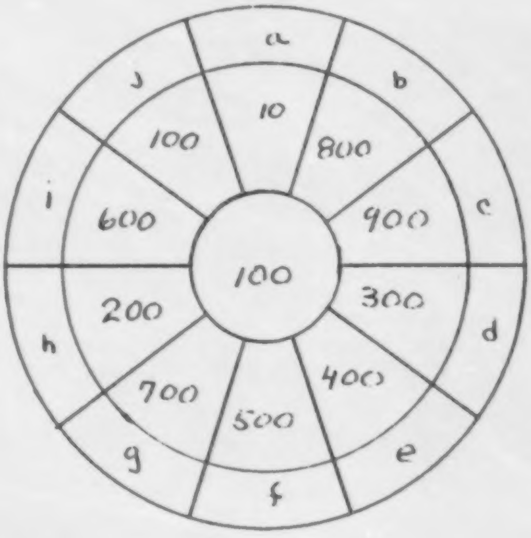
**Concept:** Multiplication of whole numbers

**Stage of Development:** Semiabstract

**Equipment:** Pizza rounds, clothepins or number cards

**Activity:** Find the product that would replace each letter in the multiplication wheel.


a
b
c
d
e
f
g
h
i
j
80
80
70
20
10
90
50
40
0
30
60


a
b
c
d
e
f
g
h
i
j
100
10
800
900
300
400
500
700
200
600
100

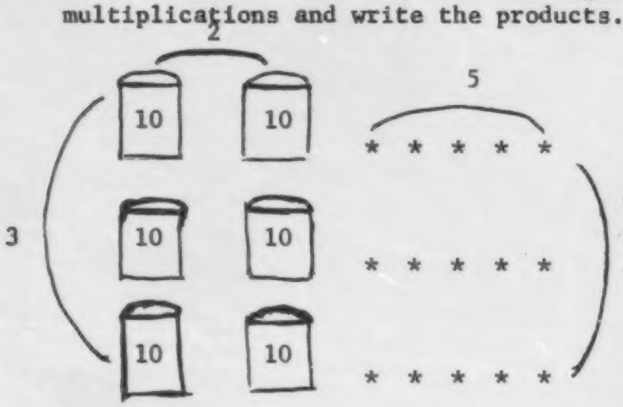
+ + + + + + + + + + + + + + +

**Concept:** Multiplication of whole numbers

**Stage of Development:** Concrete

**Equipment:** Cans and objects

**Activity:** The students can use the cans and objects to show multiplications and write the products.



| | Tens | Ones |
|---|---|---|
| X | 2 | 5 |
| 3 | | 3 |
| | 7 | 5 |

+ + + + + + + + + + + + + + + +

## DIVISION OF WHOLE NUMBERS

### Introduction

Traditionally, the most difficult skill to teach children (or adults) in the realm of computation is the process of long division. When you analyze the prerequisite skills, you can see why your students find it is so difficult.

They need to know: the addition facts, the subtraction facts, the multiplication facts, the division facts, the concept of renaming, the concept of place value, the concept of rounding off and the concept of estimating. All of the above skills are needed to successfully solve long division problems, but the two that retards the students understanding the most (if they are not understood) are the concepts of place value and renaming. The teacher should not begin the study of long division until the students have a firm understanding of these two concepts.

Note: Students will soon learn that dividing by 10 or 100 can be accomplished by merely adjusting the decimal point of the dividend. The decimal point is moved to the left one place when 10 is the divisor and two places when 100 is the divisor.

**Concept:** Division of Whole Numbers

**Stage of Development:** Concrete

**Equipment:** Coffee stirrers, Popsicle sticks, counting sticks, etc.

**Activity #1:** Use a rubber band to bundle the sticks into sets of ten. Make sure that each student has approximately 250 sticks. Some of the sticks should be left as single sticks that can be used to represent ones.

**NOTE!** MAKE SURE THAT THE STUDENTS UNDERSTAND THE CONCEPTS OF PLACE VALUE AND RENAMING (REGROUPING) BEFORE YOU BEGIN THE STUDY OF LONG DIVISION.

Each student should form a set of 36 sticks. They should have three bundles of ten and six single sticks (3 tens and 6 ones). The students are to determine, by using the sticks, how many "whole" sticks can be evenly divided among three boys. You can stimulate the students interest more if the problems are stated in terms of dividing bubble gum, marbles, candy bars or suckers and letting the sticks represent the above.

This first activity should only be concerned with dividing a certain number of sticks (bubble gum, candy, etc.) evenly among a certain number of students. Use only problems involving two digit dividends and one digit divisors that have no remainders. (e.g. 48 sticks among 4 students, 55 sticks among 5 students, etc.)

The purpose of this first exercise is to get the students acquainted with the process of partitioning the sticks into equal sets. For this reason, use problems where the tens and ones can be partitioned without involving regrouping. (e.g. Do not use problems such as 37 sticks among two students or 41 sticks among three students). Give the students sufficient problems so they can form a basic understanding of partitioning.

**Activity #2:** Give each student several bundles of ten sticks and single sticks. Verbalize a division problem that involves a two digit dividend and a one digit divisor that has no remainder, but does involve regrouping. An example of such a problem might be: Divide seven bundles and two single sticks evenly among six people or divide four tens and two ones (pieces of candy) evenly among three girls. These problems can be written on the chalkboard as $6\overline{)7 \text{ tens } 2 \text{ ones}}$ and $3\overline{)4 \text{ tens } 2 \text{ ones}}$. The student responses should be given orally for the first few problems. Later in the activity, the answer found by using the sticks should be placed above the problem in the appropriate places. *That is, the number of tens each person receives should be placed above tens and the number of ones above the ones.

$$\begin{array}{l} \quad\ 1 \qquad\quad\ 2 \\ 6\overline{)7 \text{ tens } 2 \text{ ones}} \end{array} \quad \text{and} \quad \begin{array}{l} \quad\ 1 \qquad\quad\ 4 \\ 3\overline{)4 \text{ tens } 2 \text{ ones}} \end{array}$$

Merely record the answer, do not show the division algorithm to the students at this time.

**Activity #3:** Give each student several bundles and single sticks. Write a division problem on the chalkboard that involves a two digit dividend and one digit divisor that has a remainder. (e.g. $4\overline{)6 \text{ tens } 2 \text{ ones}}$ ). The students should make up some kind of story that would be representative of the problem. Four boys had six tens and two ones (candy bars) that they would like to divide equally among them. The problem should be solved by using sticks. The results should be recorded as follows:

$$\begin{array}{l} \quad\ 1 \qquad\quad\ 5 \qquad\quad \text{R } 2 \\ 4\overline{)6 \text{ tens } 2 \text{ ones}} \end{array}$$

This means that each boy would receive fifteen candy bars with two bars left over. Repeat with other examples.

**Activity #4:** The following questioning technique should be utilized throughout the study of the concept of long division.

Example: $3\overline{)4 \text{ tens } 8 \text{ ones}}$

Question: What is the first thing we divide among the three people?
Answer: Tens

Question: How many tens will each receive?
Answer: One

Question: Where do we place this answer?
Answer: Above the tens place because we are partitioning tens.

$$\begin{array}{l} \quad\ 1 \\ 3\overline{)4 \text{ tens } 8 \text{ ones}} \end{array}$$

Question: How many tens are used?
Answer: Three

$$\begin{array}{l} \quad\ 1 \\ 3\overline{)4 \text{ tens } 8 \text{ ones}} \\ \quad\ \underline{3 \qquad\qquad\qquad} \end{array}$$

| | |
|---|---|
| Question: | How many tens do we have left? |
| Answer: | One |

$$\begin{array}{rll}
 & 1 & \\ \cline{2-3}
3 \big) & 4 \text{ tens} & 8 \text{ ones} \\
 & 3 & \\ \cline{2-3}
 & 1 &
\end{array}$$

| | |
|---|---|
| Question: | Can we give any more whole groups of tens to the three people? |
| Answer: | No |
| Question: | What do we do with the group of ten? |
| Answer: | We break it up into ones. |
| Question: | How many ones do we have all together? |
| Answer: | Eighteen |

$$\begin{array}{rll}
 & 1 & \\ \cline{2-3}
3 \big) & 4 \text{ tens} & 8 \text{ ones} \\
 & 3 & \\ \cline{2-3}
 & 1 & 8
\end{array}$$

| | |
|---|---|
| Question: | How many of these ones should each person receive? |
| Answer: | Six |
| Question: | Where do we record this answer? |
| Answer: | Above the ones place because we are partitioning the ones. |

$$\begin{array}{rll}
 & 1 & 6 \\ \cline{2-3}
3 \big) & 4 \text{ tens} & 8 \text{ ones} \\
 & 3 & \\ \cline{2-3}
 & 1 & 8
\end{array}$$

| | |
|---|---|
| Question: | How many ones did we use? |
| Answer: | Eighteen |

$$\begin{array}{rll}
 & 1 & 6 \\ \cline{2-3}
3 \big) & 4 \text{ tens} & 8 \text{ ones} \\
 & 3 & \\ \cline{2-3}
 & 1 & 8 \\
 & 1 & 8 \\ \cline{2-3}
 & & 0
\end{array}$$

| | |
|---|---|
| Question: | How many ones are left? |
| Answer: | None |
| Question: | What does the answer tell us? |
| Answer: | Each person will receive sixteen sticks. |

THIS SEQUENCE OF QUESTIONS SHOULD BE USED THROUGHOUT THE STUDY OF LONG DIVISION.

When working with the algorithm, the students will soon begin omitting the words hundreds, tens and ones from their work.

+ + + + + + + + + + + + + + +

**Concept:** Three digit dividend, one digit divisor, no renaming, no remainders.

**Stage of Development:** Concrete

**Equipment:** Sticks bundled into hundreds, tens and ones

**Activity #5:** Make up a story problem involving a division problem with a three digit dividend, one digit divisor and no remainder. Example, there are 484 cows needing to be herded into 4 cattle pens. How many cows should be placed into each pen. The algorithm would look like this:

```
    ________________________
4  /4 hundreds 8 tens 4 ones
```

If the student is going to represent the problem with sticks he will need 4 bundles of hundreds, 8 bundles of tens and 4 single sticks. Be sure and use the questioning technique discussed in Activity #4. The student would first partition the bundle of hundreds into four groups. Each group would get one bundle.

```
    1
    ________________________
4  /4 hundreds 8 tens 4 ones
    4
    ________________________
    0          8
```

The student would then partition the bundles of tens into four groups. Each group would get two bundles.

```
    1          2
    ________________________
4  /4 hundreds 8 tens 4 ones
    4
    ________________________
    0          8
               8
    ________________________
               0      4
```

The last thing the student would do is to partition the single sticks into four groups. Each group would get one stick.

```
    1          2      1
    ________________________
4  /4 hundreds 8 tens 4 ones
    4
    ________________________
               8
               8
    ________________________
               0      4
                      4
    ________________________
                      0
```

The answer would be 121 cows should be placed in each pen.

+ + + + + + + + + + + + + + +

**Concept:** Three digit dividend, one digit divisor, with renaming

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

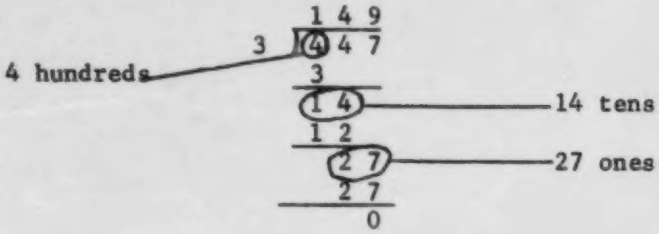
**Activity #6:** Write a division problem on the chalkboard. Use a problem which has a three digit dividend, one digit divisor which involves the process of renaming.

Example: $3\overline{)\text{4 hundreds 4 tens 7 ones}}$

At this point you may want to drop the number words ( $3\overline{)447}$ ). Four hundred forty seven books are to be distributed to three schools, would be a possible verbal problem for the algorithm. Each student should write the problem on his chalkboard. The following question-answer thought process should be followed for the problem.

1. Question: How many hundreds will each school get?
   Answer: One
2. Question: Are there any hundreds left?
   Answer: Yes - one
3. Question: How many tens will this make?
   Answer: Ten
4. Question: How many tens do you have altogether?
   Answer: Fourteen
5. Question: How many tens will each school get?
   Answer: Four
6. Question: Are there any tens left?
   Answer: Yes - two
7. Question: How many ones will this make?
   Answer: Twenty-seven
8. Question: How many ones will each school get?
   Answer: Nine
9. Question: Are there any ones left?
   Answer: No
10. Question: How many books will each school receive?
    Answer: One hundred forty nine

The completed algorithm would look like:



**NOTE!** When recording the quotient make sure the number of hundreds, tens and ones are placed over the appropriate places.

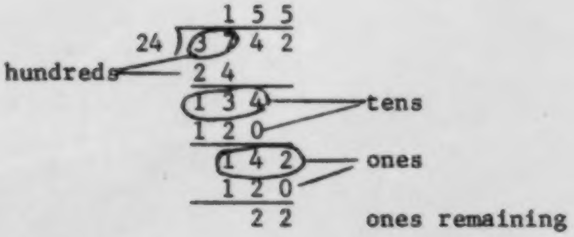
At this point in the study of long division the students should be ready to give up the sticks and only work with the algorithm. They should also be able to solve problems involving two or three digit division.

**Concept:** Division problems involving multi digit divisors and dividends.

**Activity #7:** Place a problem on the chalkboard for the students to solve. The students could use paper and pencil or individual chalkboards on which to work their problems.

Example of one problem:

24 ) 3,742 The students could think of this as 3,742 cars to be parked into twenty four parking lots.



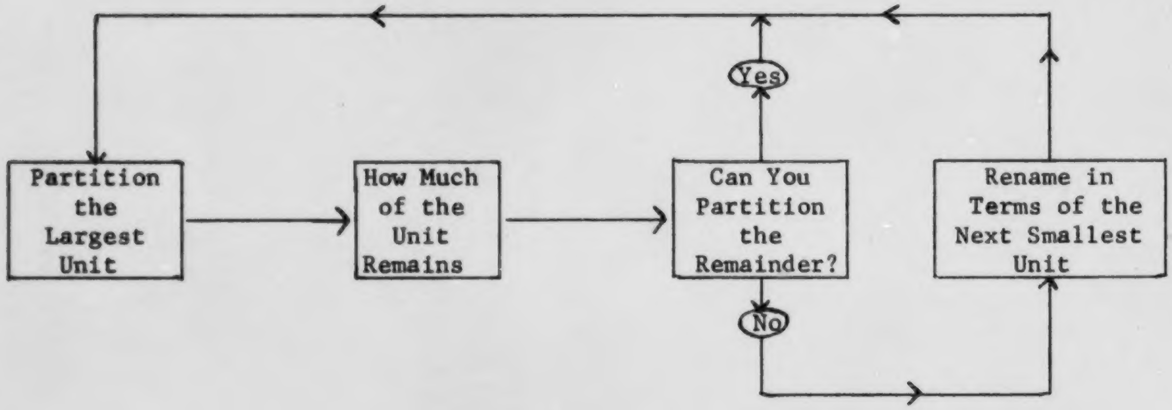
Reasoning pattern for 24 ) 3,742

The student tries to partition the thousands, can't, so renames the three thousands and seven hundreds to thirty seven hundreds → partitions the hundreds so that one group of one hundred cars will be in each lot → determines that thirteen hundreds remains → tries to partition these into twenty four lots, cannot →

renames thirteen hundreds and four tens to one hundred thirty four tens → partitions the tens so that five groups of ten cars will be placed in each lot → determines that fourteen tens remains → tries to partition these into twenty four lots, cannot → renames fourteen tens, two ones to one hundred forty two ones → partitions these ones so that five single cars are placed in each lot → determines that twenty two ones remains → tries to partition, cannot, therefore concludes that one hundred fifty five cars should be placed in each lot with twenty two cars left on the street.

+ + + + + + + + + + + + + + +

FLOW CHART FOR LONG DIVISION
(Partitioning Sets)


Partition the Largest Unit
How Much of the Unit Remains
Can You Partition the Remainder?
Yes
No
Rename in Terms of the Next Smallest Unit

# FRACTIONS

## Introduction

A fraction is a numeral. The number named by a fraction is called a fractional number.

If a set of objects is separated into two equivalent subsets, either of the subsets can be compared with the original set by saying it is one half of the original set. A set of eight objects can be separated into two subsets of four objects each, and each subset is considered to be half of the original set. Similarly, if a set of objects is separated into four equivalent subsets, each of the subsets is considered to be one fourth of the original set.

If a region is separated into two parts of the same size, then each part is one half of the original region. If a region is separated into three parts of the same size, then each part is one third of the original region, etc.

Note: The concepts of Renaming in Simplist Form, Renaming in Equivalent Form and Mixed Numerals ↔ Fractions are critical to the child's understanding of computation involving fractions and/or mixed numerals. Make sure your students understand these concepts before you introduce them to computation involving fractions.

Concept: Fractions - sets

Stage of Development: Concrete

Equipment: Counting objects (blocks, discs, sticks, etc.)

Activity: Each child receives a set of counting objects (e.g., 12). He is asked to show 1/2 of them. If he is correct, he will form a set of six objects, as this is one of the two equivalent subsets of the set of twelve. If he is asked to show 1/3 of the objects, he should form three equivalent subsets and point to one of them. For the fraction 2/3, he would point to two of the three equivalent subsets. For the fraction 1/4, he would form four sets of three objects and point to one of the sets. For the fraction 3/4, he would show the teacher three of the four equivalent subsets.

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Concept: Fractions - sets

Stage of Development: Concrete

Equipment: None

Activity: An even number of children will be asked to stand in a line in front of the class. Someone is then asked to separate this set of children into two sets, where each set represents one half of the children standing. This can be repeated by using an even number of boys and then an even number of girls. The child must be able to separate the set into two equivalent subsets; then each subset will represent one half of the original set. To study the fractions 1/4 and 3/4, the teacher can choose a number of children that is a multiple of four (e.g., four, eight, twelve, etc.). The children will have to form four equivalent subsets and use one of them for 1/4 and three of them to represent 3/4. To represent the fractions 1/3 and 2/3, the child must be able to form three equivalent subsets.

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Concept: Fractions - sets

Stage of Development: Concrete

Equipment: Pegboard, pegs, rubber bands

Activity: A certain number of pegs must be placed on the pegboard (e.g., an even number for the study of 1/2, a multiple of three for the study of 1/3 and 2/3, a multiple of four for the study of 1/4 and 3/4). The children are asked to place a rubber band around 1/2, 1/3, 2/3, 1/4, or 3/4 of the pegs on the board. For 1/2, the child will form two equivalent subsets; for 1/3 or 2/3, he will form three equivalent subsets; and for 1/4 or 3/4, he will form four equivalent subsets. To represent the fraction 2/3, the child will have to place a rubber band around two of the three equivalent subsets. Three of the four equivalent subsets will have to be banded together to represent the fraction, 3/4.

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Concept: Fractions - sets

Stage of Development: Concrete

Equipment: Two cups, one for each of two children, raisins

Activity: The teacher has a set of 10 raisins in her hand. She makes sure the children know this is a whole set. She then divides the raisins between the two children and tells them they each received one half of the set of 10. Now the class chooses partners and each pair of children receives 10 raisins to divide in half. Three children can get together and divide a set into thirds. Four children can divide a set into fourths, etc.

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Concept: Fractions - sets

Stage of Development: Concrete

Equipment: Ten quiet counters in a plastic baggie for each child

Activity: Ask the children to tell how many quiet counters they have in their baggie. Ask them to choose a partner and put their counters together. Tell how many counters they have then. At this point, they divide the counters equally and tell how many counters they have then. Teacher tells them they have one half of what they had when the two baggies were together.

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Concept: Fractions - sets

Stage of Development: Semiconcrete

Equipment: Felt stick men made out of felt and popsicle sticks (4 for each child), 20 quiet counters for each child, 4 felt men and 20 counters for teacher (teachers felt men and counters have magnetic strip on the back), magnetic board

Activity: The teacher shows two felt men on the magnetic board and asks the children to put two felt men in front of them. The teacher then chooses two quiet counters to divide between the men -- each man receives one half of the whole set of quiet counters. Go ahead now and have the children use 4, 6, 8, 10, 12, etc. quiet counters. Now allow the children to use three felt men and divide the counters equally. This time, the child should discover 3, 6, 9, 12, etc. Go ahead and use four felt men to divide the sets equally the next time.

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Concept: Fractions - sets

Stage of Development: Semiconcrete

Equipment: Flannel boards, flannel objects, magnetic board, magnetic objects

Activity: Several objects are placed on one of the above boards (an even number when studying 1/2, a multiple of three when studying 1/3 or 2/3, a multiple of four when studying the fractions 1/4 and 3/4). One child is then asked to separate the set into equivalent subsets (two when studying 1/2, three when studying 1/3 or 2/3, four when studying 1/4 or 3/4). Next, he shows the teacher a set that represents one of the above fractions (one of the two equivalent sets for 1/3, two of the three equivalent sets for 2/3, one of the four equivalent sets for 1/4, three of the four equivalent sets for 3/4).

+ + + + + + + + + + + + + + + +

**Concept:** Fractions - sets

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, transparencies with regions divided into halves, thirds and fourths

**Activity:** Give the children the 20 quiet counters and allow them to divide them equally as many ways as they can on the transparency. They then record the whole number of objects and the number of objects that are in each fractional part.

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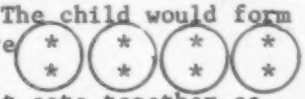
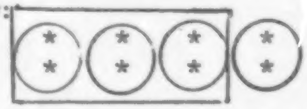
**Concept:** Fractions - sets

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, pennies or other objects that can be used on the projector

**Activity:** A certain amount of objects are placed on the overhead projector (e.g., an even number if 1/2 is the fraction studied, a multiple of three for the study of the fractions 1/3 and 2/3, a multiple of four for the study of the fractions 1/4 and 3/4). Someone is then asked to separate the objects into equivalent subsets, and to show a set that represents a fraction that the teacher has given him. (e.g., the student is given the following objects, * * * *
* * * *
with the instructions that he is to form a set that represents 3/4 of the objects.) The child would form four equivalent sets as shown here  and he would then form a set by grouping three of these equivalent sets together as shown below:

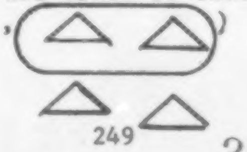
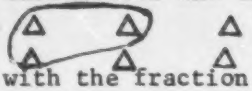


+ + + + + + + + + + + + + + + +

**Concept:** Fractions - sets

**Stage of Development:** Semiabstract

**Equipment:** Individual chalkboards

**Activity:** The children are told to draw a set of objects on their chalkboards (e.g., four traiangles). Then, they are asked to enclose one half of them in a set (e.g.,  ) Next, they are asked to draw six triangles on their chalkboards and enclose half of them in a set (e.g.,  ).

As long as they are working with the fraction 1/2, the children will be asked to draw an even number of objects. A multiple of three is chosen when working with the fractions 1/3 and 2/3. A multiple of four is used for the fractions 1/4 and 3/4.

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**Concept:** Fractions - sets

**Stage of Development:** Abstract

**Equipment:** Pencil and paper

**Activity:** Arrange different sets on the paper and have the children show 1/2, 1/3, or 1/4 of the whole number of objects.

+ + + + + + + + + + + + + + +

**Concept:** Fractions - regions

**Stage of Development:** Concrete

**Equipment:** Construction paper circles

**Activity:** The teacher shows the child how to fold a circle in half. Each of the children then fold their circle in half. Teacher folds another circle in fourths. The children then fold their circle in fourths. This can be done with circles, squares, rectangles and triangles. After they have folded the construction paper pieces, they may want to cut them and compare the pieces.

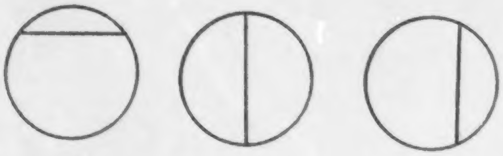
+ + + + + + + + + + + + + + +

**Concept:** Fractions - regions

**Stage of Development:** Concrete

**Equipment:** Small boards

**Activity:** Boards that have been cut into two parts are arranged on a table in front of the class. Only one board should have been cut into parts that are equal in length. A child is asked to choose the board that has been cut in half. The teacher should stress the fact that the parts must be of the same magnitude before one can say that the board has been cut in half. The same thing can be done with other shapes. For example, circles can be sectioned as shown below:

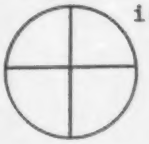
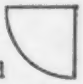


+ + + + + + + + + + + + + + + +

Concept: Fractions - regions

Stage of Development: Semiconcrete

Equipment: Flannel board, flannel regions cut into equal parts, magnetic board, regions cut into equal parts (with small magnets on their backs)

Activity: One of the regions that has been cut into equal parts should be placed either on the flannel board or the magnetic board. The teacher states a fraction. A child is then asked to show the portion(s) of the region that represents this fraction. (e.g.,  is the region, 1/4 is the fraction, and  is the portion that represents the fraction) This can be repeated with other regions and fractions.

+ + + + + + + + + + + + + + + +

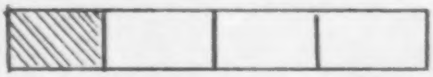
Concept: Fractions - regions

Stage of Development: Semiabstract

Equipment: Overhead projector, transparencies showing regions that have been divided into equal parts, overhead projector pen

Activity: The teacher will prepare several transparencies, showing regions that have been divided into equal parts. For the study of the fraction 1/2, the region should be divided into an even number of equal parts (two, four, six, eight, etc.). For the study of the fractions 2/3 and 1/3, the region is divided into a multiple of three (three, six, nine, etc.) equal parts. For the study of the fractions

1/4 and 3/4, the region is divided into a multiple of four (four, eight, twelve, etc.) equal parts. The teacher states a fraction (e.g., one fourth). A child then shades in the region(s), that represent the fraction for the above example.



+ + + + + + + + + + + + + + +

Concept: Fractions - regions

Stage of Development: Concrete

Equipment: Construction paper circles for each child

Activity: The teacher folds her circle in half and tells the children each part is 1/2. The children then fold their circle in half too. This continues with fourths and thirds. The children can place a block on 1/3, 2/3, 1/4 or whatever fractional part the teacher verbalizes.

+ + + + + + + + + + + + + + +

Concept: Fractions (1/2)

Stage of Development: Concrete

Equipment: Many magazine pictures

Activity: Have the children fold the pictures in half to see if one side of the picture is exactly like the other side. Each part is called 1/2.

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Concept: Fractions (1/2)

Stage of Development: Concrete

Equipment: Construction paper and scissors

Activity: Direct the child to fold the construction paper in half. He then cuts a design on the fold. When he opens it up, one side is exactly like the other.

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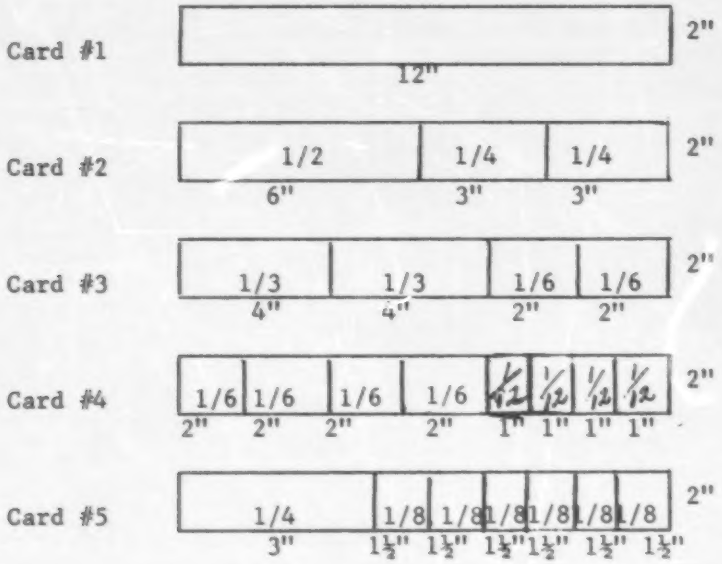
**Concept:** Fractional parts of one

**Stage of Development:** Concrete

**Equipment:** Fraction cards, rectangle shapes 2" x 12" labeled as follows:


Card #1
2"
12"
Card #2
1/2
1/4
1/4
2"
6"
3"
3"
Card #3
1/3
1/3
1/6
1/6
2"
4"
4"
2"
2"
Card #4
1/6
1/6
1/6
1/6
1/12
1/12
1/12
1/12
2"
2"
2"
2"
2"
1"
1"
1"
1"
Card #5
1/4
1/8
1/8
1/8
1/8
1/8
1/8
2"
3"
1½"
1½"
1½"
1½"
1½"
1½"

These need to be cut into their fractional parts.

**Activity:** All of the fractional parts are put in a pile. The six children take turns drawing fractional parts from the pile. The first player to put together three "ones" wins the game.

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**Concept:** Fractional parts of one

**Stage of Development:** Concrete

**Equipment:** Fractional parts include several 1/2, 1/4, 1/3, 1/6, 1/12, 1/8 sections

**Activity:** The three or four players take turns drawing fractional parts from the pile until each player has 10 parts. The parts left in the pile are removed and not used in the game. The players try to make as many whole "ones" as possible. The first player to fit all his parts into one or more "ones" is winner. If no one can use all his parts, each player, in turn, may then ask any other player for the parts he needs. He must give back equivalent parts. (2/6 for 1/3, 2/4 for 1/2, 2/8 for 3/12, etc.)

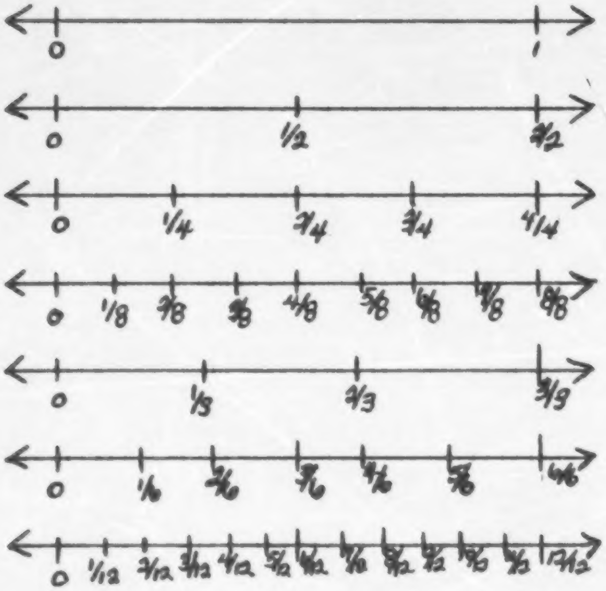
+ + + + + + + + + + + + + + + +

**Concept:** Fractions - Renaming in simplist form and equivalent form. A fraction is in its simplist form when its numerator and denominator do not share a common factor. Ex: 3/5, 7/11, 5/13 are in simplist form. 4/16, 5/30, 120/144 are not in simplist form.

**Stage of Development:** Semiabstract

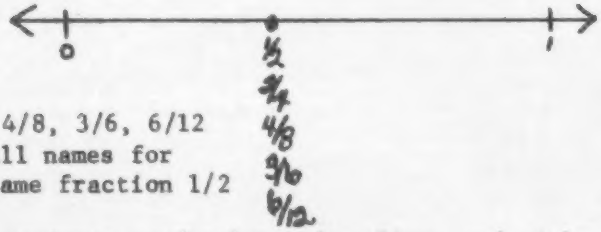
**Equipment:** Chalkboard

**Activity:** Draw several parallel number lines on the chalkboard. Label the number lines as shown below.


0 1
0 1/2 2/2
0 1/4 2/4 3/4 4/4
0 1/8 2/8 3/8 4/8 5/8 6/8 7/8 8/8
0 1/3 2/3 3/3
0 1/6 2/6 3/6 4/6 5/6 6/6
0 1/12 2/12 3/12 4/12 5/12 6/12 7/12 8/12 9/12 10/12 11/12 12/12

etc.

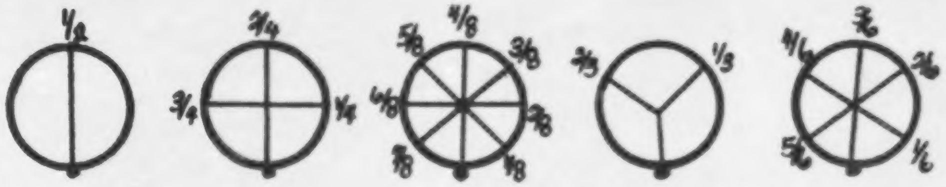
Equivalent fractions will be those fractions that would be the same point on the number line if all the above number lines were made to coincide. Ex. for 1/2


0
1
1/2
2/4
4/8
3/6
6/12

2/4, 4/8, 3/6, 6/12
are all names for
the same fraction 1/2

Have students study the number lines and pick out fractions that are equivalent. Also have them identify the fraction that is the simplist form of the fraction.

| | |
|---|---|
| Concept: | Fractions - Renmaing in simplist form and equivalent form |
| Stage of Development: | Semiabstract |
| Equipment: | 10" tagboard circles |
| Activity: | Subdivide the circles into different numbers of equivalent parts |



Make sure that all the circles have the same diameter. Students can find fractions that are equivalent by matching up the dots and finding out which fractional parts coincide. Ex. 2/8 and 1/4, 1/3 and 2/6, 1/2, 2/4, 4/8, 3/6. Students should make sure that the dots match up as well as the centers. Give several segmented circles to a group of children and have them list all the equivalent fractions they can find. When all groups are finished write their results on the chalkboard. Also call attention to the fraction that is in simplist form for each set of equivalent fractions.

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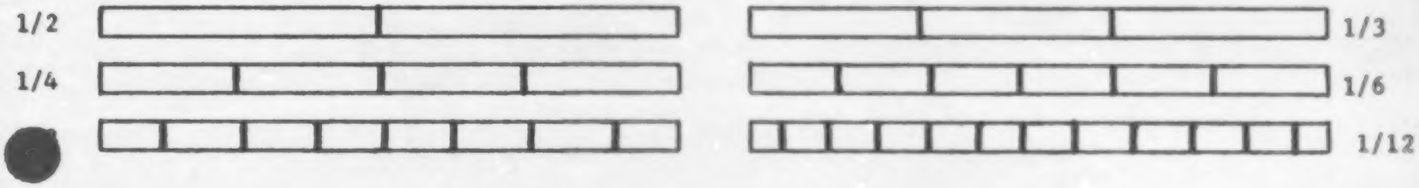
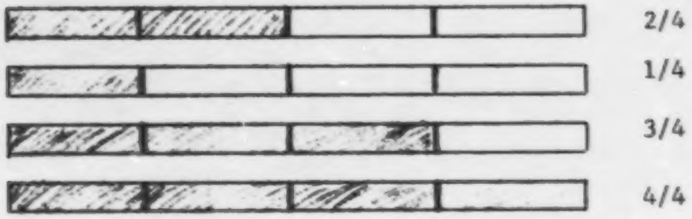
| | |
|---|---|
| Concepts: | Fractions - Renaming in simplist form and equivalent form |
| Stage of Development: | Semiabstract |
| Equipment: | Rectangular strips of tagboard (1" x 12") |
| Activity: | The strips of tagboard will represent a unit. Subdivide each of these units into the fractional parts. Halves, fourths, eights, twelfths, sixths, thirds. |

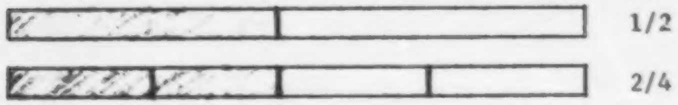


Shade in one or more parts of each subdivided strip. Ex. for 1/4.



Do the same with the other strips. In fact you may want to construct more than one strip for each shading, (ie. two or three of the 1/8 divided strips shaded exactly the same).

Give each of your students several of these shaded strips. They can determine which strips represent equivalent fractions by comparing the length of the shaded portion of the strip.



The shaded portion of the strip representing 1/2 and the strip representing 2/4 are of the same length and therefore represent fractions that are equivalent.

Ask your students to find as many equivalent fractions as possible and use the strips given to them. They are to record the equivalent fractions on their chalkboards or paper for your observation. The students should also identify which of the equivalent fractions is in simplist form.

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**Concept:** Fractions - Equivalent Form

**Stage of Development:** Semiabstract

**Equipment:** Concentration Game

**Activity:** Load the Concentration Game board with various equivalent fractions, ex., 1/2, 2/4, 3/6, 16/32, 2/3, 6/9, etc. Play the game as it is usually played, except the object of the game will be to match two fractions that are equivalent. The winner will be the team or the individual that gets the most equivalent fraction.

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**Concept:** Fractions - Equivalent Forms

**Stage of Development:** Semiabstract

**Equipment:** Could Be Game

**Activity:** On one side of the Could Be card mark a fraction in its simplist form. On the other side of the card write one of its equivalent forms, ex. on one side write $\boxed{\frac{3}{5}}$ on the other side $\boxed{\frac{21}{35}}$

To play the game expose the fraction in simplist form to the students. The student has to tell which equivalent fraction is on the back of the card. This should bring about a lively discussion of the possible equivalent forms of the fraction. The first student to correctly name the equivalent fraction gets the card. The student with the most cards at the conclusion of the contest is declared the winner.

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**Concept:** Fractions - Mixed Numeral ⟷ Fractions

Giving mixed numerals, fraction names is of importance when mixed numerals are involved in computation. A mixed numeral has been renamed as a fraction when it no longer contains a whole number part, ex. 3 1/5 renamed as 16/5, 5 2/7 renamed as 37/7. The process of renaming mixed numerals to fractions is shown below: $5\ 2/7 \leftrightarrow 5 + 2/7 \leftrightarrow 35/7 + 2/7 \leftrightarrow 37/7$. What it amounts to is renaming the whole number part of the mixed numeral as a fraction with the same denominator, then adding the numerator of the two fractions. Students should be given many opportunities to practice this renaming process. Your brighter students will soon discover a short cut to the renaming process. The short cut method is shown below:

$$6\frac{2}{5} = \frac{(6 \times 5) + 2}{5} = \frac{30 + 2}{5} = \frac{32}{5}.$$

The student multiplies the denominator of the fraction times the whole number part, adds the numerator to this product and this is the new numerator for the fraction.

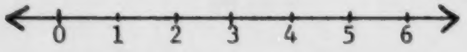
Activities: Most of your activities for changing mixed numbers to fractions will have to be done on individual chalkboards or on paper. The game of Concentration could be played by matching mixed numbers with their fraction names.

Concept: Fractions - Order on the Number Line

Stage of Development: Semiabstract

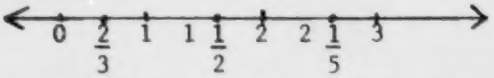
Equipment: Chalkboard

Activity: Draw a number line on the chalkboard. Label the whole number divisions of the line.



etc.

Write several fractions and mixed numerals on the chalkboard. The students are to label points on the number line that would be representative of the given fractions and mixed numbers



etc.

The same procedure could be done on a overhead projector number line transparency.

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# ADDITION OF FRACTIONS

## Introduction

Abstractly, the definition for adding two fractions together is

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

when a, b, c, d are whole numbers.

The definition in narrative form would read: Rename the fractions so they both have the same denominators, add the numerator. Use this sum as the numerator over the common denomination making a fraction which is the answer, ex.

$$\frac{3}{5} + \frac{1}{3} = \frac{9}{15} + \frac{5}{15} = \frac{9 + 5}{15} = \frac{14}{15}.$$

NOTE: When you get to the stage of reinforcement for the skill of adding fractions, use many of the same activities you used for addition of whole numbers. Change the problems to fractions.

To add mixed numerals use the same procedure as with fractions. Rename the fractional parts so they have the same denominator, then add the whole number parts as well as the fractional parts. Simplify the answer.

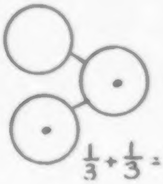
**Concept:** Addition of fractions -- like denominators with sums less than one

**Stage of Development:** Concrete

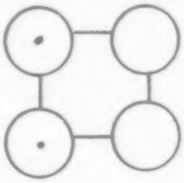
**Equipment:** Plastic can carrier cut into three, four, five or six spaces, counters

**Activity:** Each child places a counter into a space. Name the space filled (as part of the whole). Fill another space. Name the sum. Rename if applicable.

e.g.



$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$



$\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$ or $\frac{1}{2}$

+ + + + + + + + + + + + + + + +

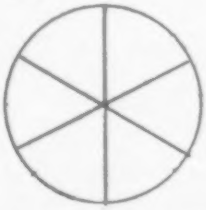
**Concept:** Addition of fractions -- like denominators with sums less than one

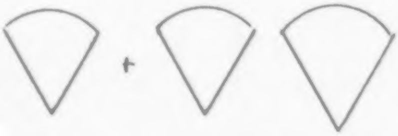
**Stage of Development:** Concrete

**Equipment:** Paper plates divided into 1/3, 1/4, 1/5, 1/6

**Activity:** Each child places a piece on the surface. Name the piece (as part of the whole). Add another piece. Name the sum. Rename if applicable.

e.g.





$\frac{1}{6}+\frac{2}{6}=\frac{3}{6}$ or $\frac{1}{2}$

+ + + + + + + + + + + + + + + +

**Concept:** Addition of fractions -- like denominators with sums less than one

**Stage of Development:** Concrete

**Equipment:** Plastic modeling clay or play-dough and string

**Activity:** Each child makes a "pancake", square, rectangle, etc. Divide with string into desired sections. Name the pieces (as parts of the whole). Take a piece -- add more pieces -- name the sum. Rename if applicable.

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**Concept:** Addition of fractions -- like denominators with sums less than one

**Stage of Development:** Concrete

**Equipment:** Clear plastic cups marked 1/4, 1/2, 3/4, and 1 (Use your measuring cup to find the lines -- mark with permanent black marker.)

**Activity:** Fill cups to 1/4. Pour into an unmarked cup. Add another 1/4 cup. Pour back the first cup. Ask why it is not named as 2/4. Do same for 4/4 as 1.

+ + + + + + + + + + + + + + + +

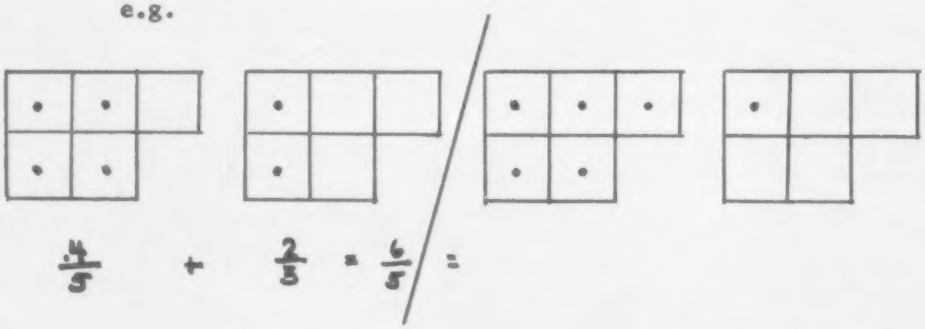
**Concept:** Addition of fractions -- like denominators with sums greater than one

**Stage of Development:** Concrete

**Equipment:** Egg cartons cut into 3, 4, 5, 6, 12 and counters (enough for two of each size for each child)

**Activity:** Determine two fractional numbers whose sum is greater than one. Place one number in one container, the other in the second container. Name the sum. Rename the sum.

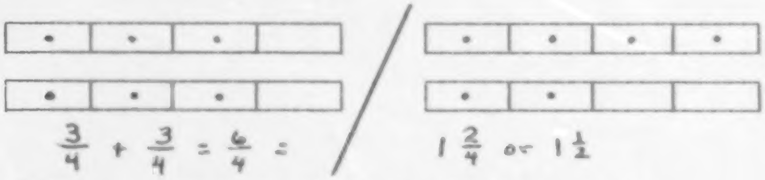
e.g.



$$\frac{4}{5} + \frac{2}{5} = \frac{6}{5} =$$

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Addition of fractions -- like denominators with sums greater than one |
| Stage of Development: | Concrete |
| Equipment: | Strips divided into sections and counters (enough for two strips for each child) |
| Activity: | |



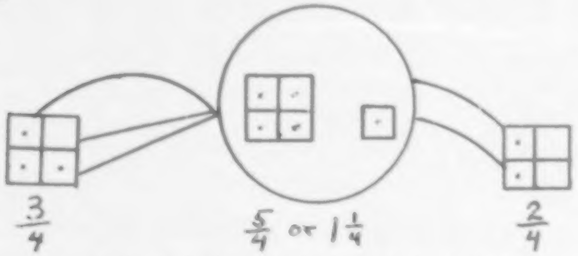
$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} =$
$1\frac{2}{4}$ or $1\frac{1}{2}$

NOTE: You don't "reduce" the fraction -- it remains the same size. You RENAME.

+ + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Addition of fractions -- like denominators with sums greater than one |
| Stage of Development: | Concrete |
| Equipment: | Blocks and paper plates (three plates per child) |
| Activity: | Arrange blocks in two like configurations. Place one fractional part from the left into the center plate. Place one fractional number from the right into the center plate. Form the original configuration in the center plate. |

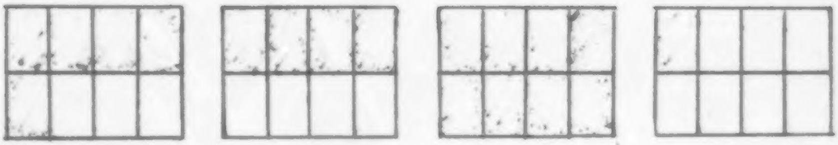
e.g.



+ + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Addition of fractions -- like denominators with sums greater than one |
| Stage of Development: | Semiconcrete and/or semiabstract |
| Equipment: | Paper grids with magnets or transparency grids and projector pin (Four of each grid) |
| Activity: | Cover or shade fractional parts on two identical grids. Find the sum by color or shading on extra two grids. |

e.g.



$\frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$

NOTE: Ability to determine equality and inequality of fractions is a prerequisite.

+ + + + + + + + + + + + + + + +

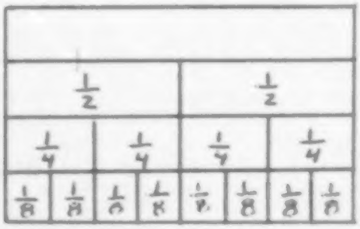
**Concept:** Addition of fractions -- unlike denominators (first review renaming)

**Stage of Development:** Concrete

**Equipment:** Cuisenaire rods -- brown (8), dark green (4), red (2), and white (1)

**Activity:** Choose only those fractions where one must be renamed.

e.g.



$\frac{1}{2} + \frac{1}{4}$

$\frac{1}{4} + \frac{1}{8}$

$\frac{1}{2} + \frac{1}{8}$

+ + + + + + + + + + + + + + + +

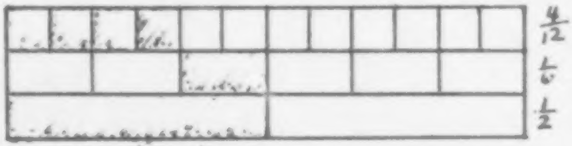
**Concept:** Addition of fractions -- unlike denominators (first review renaming)

**Stage of Development:** Concrete

**Equipment:** Fraction bars (commercial set available by request)

**Activity:** Choose fractions for addition that require renaming and the denominator is 2, 3, 4, 6, or 12.

e.g.,



$4/12 + 1/6 = 1/2$

+ + + + + + + + + + + + + + + +

**Concept:** Addition of fractions -- unlike denominators (first review renaming)

**Stage of Development:** Semiconcrete/semiabstract

**Equipment:** Use colored rods on transparency paper for overhead or with magnets on magnetic board

**Activity:** Choose fractions for addition that require renaming and the denominator is 2, 3, 4, 6, or 12.

NOTE: Ability to determine quality and inequality of fractions is a prerequisite.

+ + + + + + + + + + + + + + + +

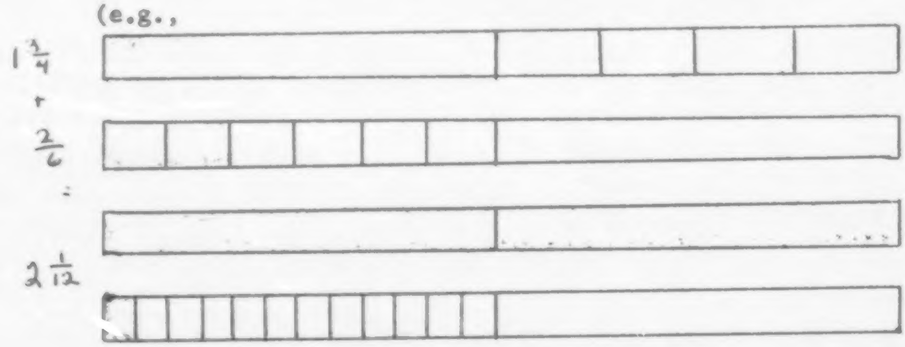
**Concept:** Addition of fractions -- mixed numerals

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** If necessary, show inequality of first bar (fourths) and second bar (sixths). Rename to last bar (twelfths).

(e.g.,



NOTE: The only drawback to using fractional circles or squares to do this type of problem is the difficulty of comparison. The bars can be easily compared by lining up one beneath the other using a different color for each bar.

+ + + + + + + + + + + + + + + +

**Concept:** Addition of fractions - mixed numerals

**Stage of Development:** Semiconcrete/semiabstract

**Equipment:** Transparency bars (for overhead) or magnetic bars (for magnetic boards)

**Activity:** Same as above.

+ + + + + + + + + + + + + + + +

# SUBTRACTION OF FRACTIONS

## Introduction

The formal definition of the subtraction of fractions is

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

when a, b, c and d are whole numbers.

In laymens terms this would mean, rename the fractions so they both have the same denominator, then find the answers by subtracting the numerators. ex.

$$\frac{7}{8} - \frac{2}{3} = \frac{21}{24} - \frac{16}{24} = \frac{21 - 16}{24} = \frac{5}{24}$$

To subtract a fraction from a whole number, rename the whole number so it is a fraction having the same denominator as the given fraction then subtract. ex.

$$3 - \frac{3}{4} = \frac{12}{4} - \frac{3}{4} = \frac{12 - 3}{4} = \frac{9}{4} \text{ or } 2\,\frac{1}{4}$$

When subtracting mixed numerals, rename the numerals in such a way that both numerals have fractional parts which have the same denominator and the numerator of the minuend is greater than the numerator of the subtrahend. Then subtract the whole number parts and fractional parts. ex.

$$6\,\frac{2}{3} - 2\,\frac{3}{4} = 6\,\frac{8}{12} - 2\,\frac{9}{12} = 5\,\frac{20}{12} - 2\,\frac{9}{12} = 3\,\frac{11}{12}$$

Use many of the activities from the section on subtraction of whole numbers by adapting them for the reinforcement of fractions.

281

**Concept:** Subtraction of common fractions with like denominators

**Stage of Development:** Concrete

**Equipment:** Use the same materials as suggested for "Addition of fractions with like denominators"

**Activity:** Reverse the activities used in addition

+ + + + + + + + + + + + + + + +

**Concept:** Subtraction of common fractions with unlike denominators

**Stage of Development:** Concrete

**Equipment:** Use the same material as suggested for "Addition of Fractions with Unlike Denominators"

**Activity:** Reverse the activities used in addition.

+ + + + + + + + + + + + + + + +

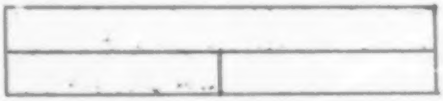
**Concept:** Whole numbers subtract common fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Line up fraction bars to prove the problem.

e.g.,



$1 - \frac{1}{2} = \frac{1}{2}$

+ + + + + + + + + + + + + + + +

**Concept:** Mixed numeral subtract common fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Reverse the activity for "Addition of mixed number and common fraction".

+ + + + + + + + + + + + + + + +

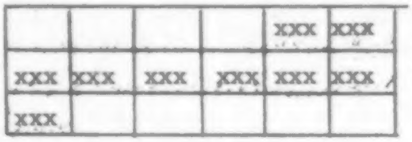
**Concept:** Mixed numeral subtract mixed numeral

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Line up fraction bars to prove the problem.

(e.g.,



xxx
xxx
xxx
xxx
xxx
xxx
xxx
xxx
xxx

$2\frac{1}{6} - 1\frac{3}{6} = \frac{4}{6}$

)

+ + + + + + + + + + + + + + +

# MULTIPLICATION AND DIVISION OF FRACTIONS

## Introduction

How did you learn to solve 2/3 x 3/4 = ☐ ? Probably, like me, you learned to multiply the numerators and divide that quantity by the product of the denominators. That is, (2 x 3) / (3 x 4) = 6/12. The logical question is, why does that procedure yield the correct result to the initial problem?

How did you learn to solve 3/4 ÷ 1/6 = ☐ ? Again, we both probably learned the clever rule, invert the second fraction and multiply it by the first fraction. Specifically, 3/4 x 6/1 = (3 x 6) / (4 x 1) = 18/4. Even more than with multiplication the question arises, why does that procedure, which seems very arbitrary, yield the answer to the initial division problem?

If we are going to teach children to multiply and divide fractions, then let's:

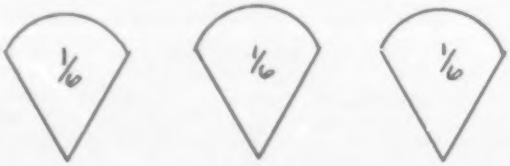
1. have them understand what multiplication and division of fractions mean by developing computational algorithms through the use of concrete materials, and
2. use what the children already know about multiplication and division of whole numbers and extend those ideas to include fractional quantities.

## FRACTIONAL NUMBERS

Before dealing with multiplication and division of fractions, we need to agree on what simple fractions mean. Look at the blue circular section. Why does it have 1/4 written on it? What do the numerals mean? The following interpretation is usually made:

$$\begin{array}{l} 1 \leftarrow \underline{\text{1 part of}} \\ \hline 4 \leftarrow \text{4 equal parts.} \end{array}$$

Pick up three brown pieces. What fraction could we assign to those three pieces combined? (Answer: 3/6)



Consider the people in this room. How could we, without multiplying, put 2/3 of the group of people in one place? We could sort the people, one by one, into three equal groups and then put two of those three equal groups (2/3) together. We will use this same idea throughout our work with the circular sections.

# MULTIPLICATION OF FRACTIONS

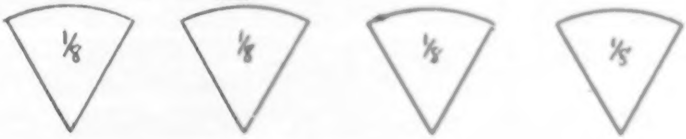
Let's use these fraction ideas to work with multiplication of fractions. We decided to build on the multiplication of whole numbers.

In the problem 3 x 4 = □ , what do the numerals 3 and 4 mean? Generally, 3 x 4 is interpreted to mean three sets of four; that, three indicates the number of sets and four indicates the size of each set.

For the multiplication of fractions, the fractional numbers will have identical meanings to their corresponding whole numbers in multiplication of whole numbers problems.

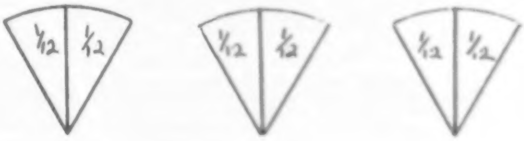
Specifically, if □ x △ = ○
□ indicates the number of sets, and
△ indicates the size of each set.

Let's try 4 x 1/8 = □



4 sets of 1/8 = 4/8

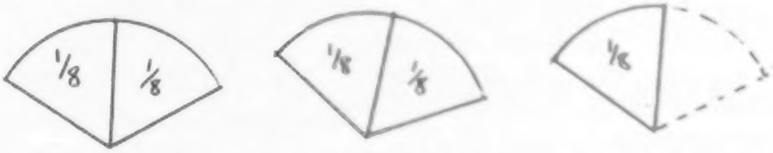
Now let's do 3 x 2/12 = □



So three sets of 2/12 = 6/12

NOTE: To work this problem, it is important to focus on the size of the set first (2/12) and then on the number of sets (3). This idea is very important for later problems.

Now let's try 2 1/2 x 2/8 = □



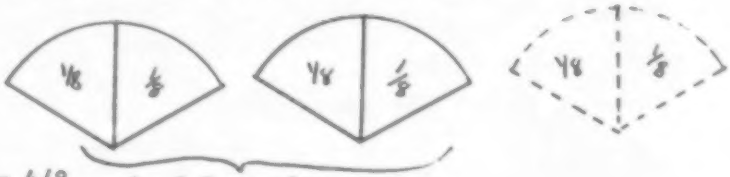
1 set of 2/8 1 set of 2/8 ½ set of 2/8

So, 2 1/2 sets of 2/8 = 5/8

**NOTE:** To find 1/2 set of 2/8 we put the 2/8 into two equal groups (1/8 in each group) and use one of those two equal groups (1/2).

Here's a different one ÷ 2/3 x 6/8 = ☐

First we take out a set of 6/8 (See note on previous page). Our problem tells us that we want 2/3 sets of 6/8. How do we find 2/3 sets? Remember how we found 2/3 of the people in this room? That's right, we put our objects (6/8) into three equal groups and use two of those three equal groups (2/3).

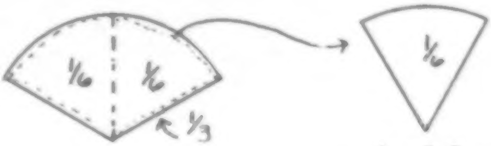


Thus, 2/3 x 6/8 = 4/8

2 of 3 equal groups

Now, let's focus on some problems that will generalize nicely to give us a computational algorithm for multiplying fractions.

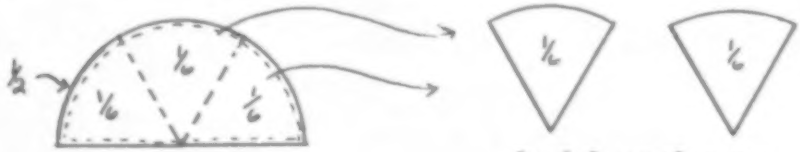
1) 1/2 x 1/3 = ☐



1 of 2 equal parts

Dividing 1/3 into two equal parts is easily done by finding two identical circular sections which together exactly cover 1/3. One of those two equal pieces then represents 1/2 of the 1/3 piece. Thus, 1/2 x 1/3 = 1/6

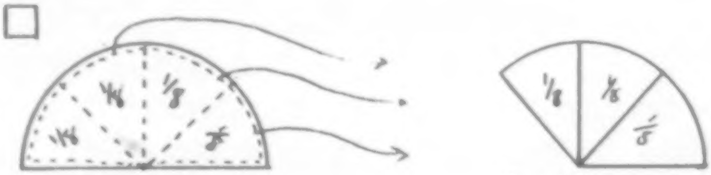
2) 2/3 x 1/2 = ☐



2 of 3 equal parts

Finding three identical pieces which together cover 1/2 and using two of those three pieces tells us that 2/3 x 1/2 = 2/6

3) 3/4 x 1/2 = ☐



Thus 3/4 x 1/2 = 3/8

3 of 4 equal parts

On the basis of these three examples it looks like

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

One logical comment might be "It doesn't look like that rule applies to the first problems we worked." However, if you apply that rule to the first problems you will find that, by using equivalent fractions (which the students will already have had), the rule works nicely for those problems also.

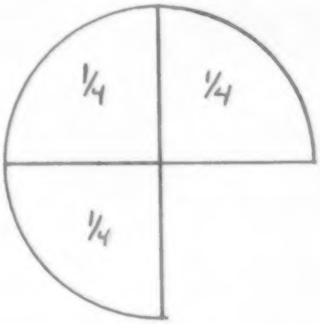
Thus, we have done what we set out to do. Namely, we have developed a computational algorithm for multiplying fractions by using concrete materials and the basic ideas of multiplying whole numbers.

## DIVISION OF FRACTIONS

In the division problem $28 \div 7 = \square$ , what do the numerals 28 and 7 mean? The most common interpretation (and the one we will use for fractions too) is "How many sevens are there in 28?" To do this problem with bottle caps, we would take 28 bottle caps and put them into sets of seven. We would find that there are four sevens in 28.

Let's try the same ideas with the circular sections.

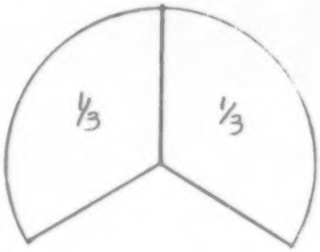
Try $3/4 \div 1/4 = \square$



The question becomes, how many fourths are there in three fourths? The answer is clearly three.

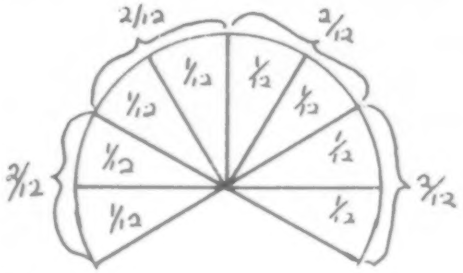
Let's do $2/3 \div 2/12 = \square$

First we take out 2/3.



Then we cover with sets of 2/12.

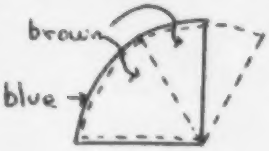
We see that there are four sets of 2/12 in 2/3. Thus, $2/3 \div 2/12 = 4$.



Now, I want you to try some problems using the circular sections--but with the numerals face down. (This is a very important exercise.)

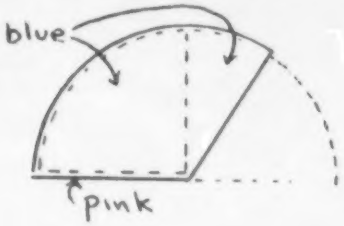
1. How many purples in a pink? (Answer four.) Translating this question into a number sentence we get $1/3 \div 1/12 = 4$

2. How many browns in a blue?



By covering, we see that there are 1 1/2 browns in a blue. Thus, our number sentence is $1/4 \div 1/6 = 1\ 1/2$

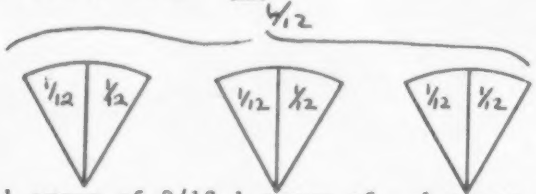
3. How many blues in a pink?



By covering, we see that there are 1 1/3 blues in a pink. Thus $1/3 \div 1/4 = 1\ 1/3$

Now, let's try some problems which will generalize nicely to give us a computational algorithm.

4. a) $6/12 \div 2/12 = \square$



1 group of 2/12 1 group of 2/12 1 group of 2/12

Now do these:

b) $6/8 \div 2/8 = \square$

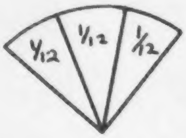
c) $6/6 \div 2/6 = \square$

In doing these problems, you notice that the procedure you follow is the same in each problem. In each case you take six pieces of the indicated size (twelfths, eighths, or sixths) and sort them into groups of two. In each case there are three groups.

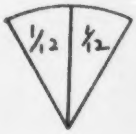
Thus, $6/12 \div 2/12 = 3$
and $6/8 \div 2/8 = 3$
and $6/6 \div 2/6 = 3$

5. Now let's try this set of problems

a) 5/12 ÷ 3/12 = ☐



1 group of 3/12



2/3 group of 3/12

Thus, 5/12 ÷ 3/12 = 1 2/3

Now do these:

b) 5/8 ÷ 3/8 = ☐

c) 5/6 ÷ 3/6 = ☐

Again you find that the procedures are identical. In each case you place five objects into groups of three and you find that you get 1 2/3 groups.

Thus, 5/8 ÷ 3/8 = 1 2/3
and 5/6 ÷ 3/6 = 1 2/3

Are you willing to generalize a rule for any problem of the form $\frac{a}{c} \div \frac{b}{c} = \square$ ?

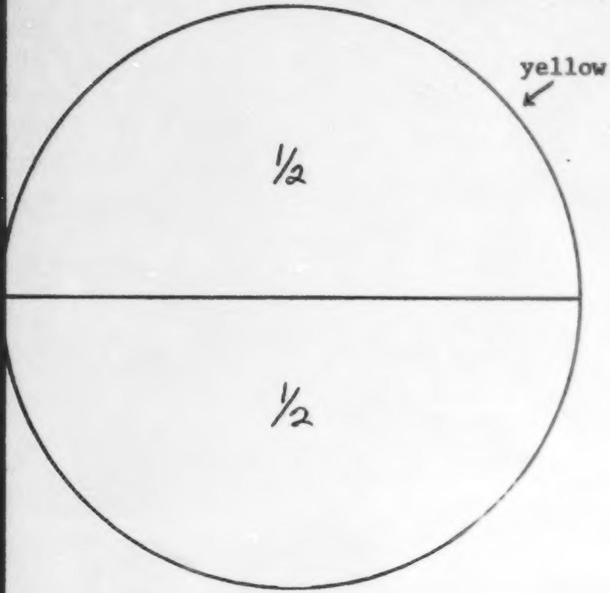
If you said $\frac{a}{c} \div \frac{b}{c} = a \div b$ or $\frac{a}{b}$ you are exactly right.

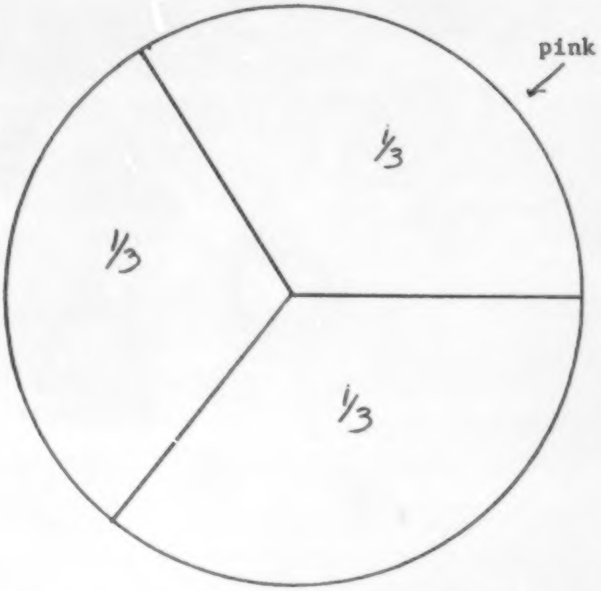
To solve the problem $\frac{a}{c} \div \frac{b}{c} = \square$ with circular parts you would take "a" of the "1/c" pieces and sort them into groups of size "b"; that is, you'd be solving the problem, how many sets of size "b" are there in "a"? This is exactly the whole number problem a ÷ b = ☐

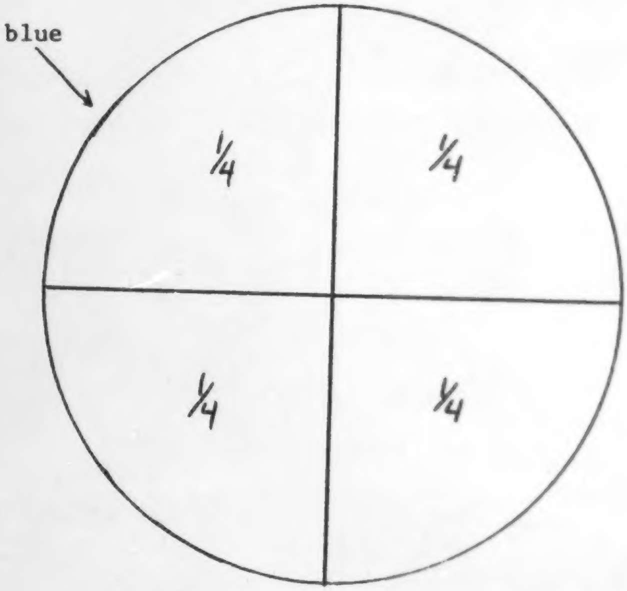
Thus, we have an easy computational algorithm for the division of like fractions. But what about division of unlike fractions? Well, since we can make any pair of fractions have like denominators (by using equivalent fractions) we have an algorithm for dividing any pair of fractions--just make the fractions have like denominators and divide the numerator of the first by the numerator of the second.

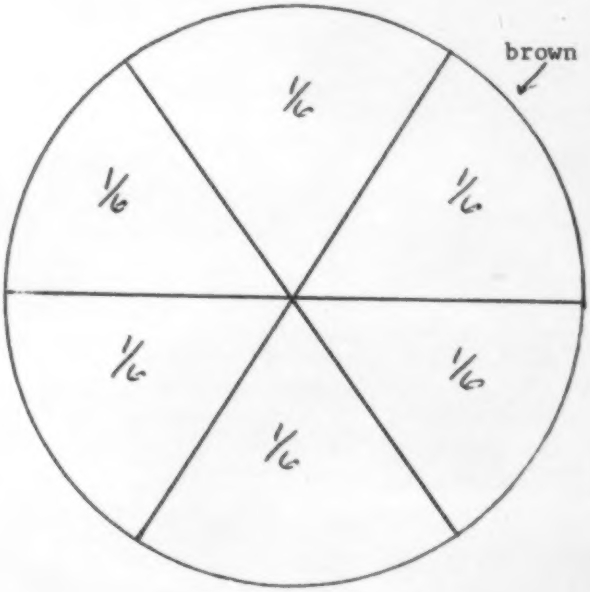
This method of dividing fractions has much going for it. Most important, the computational algorithm can be developed through the use of concrete materials by using the basic ideas of division of whole numbers. There is no comparable way to provide a physical representation for the rule you and I learned for dividing fractions--invert and multiply.

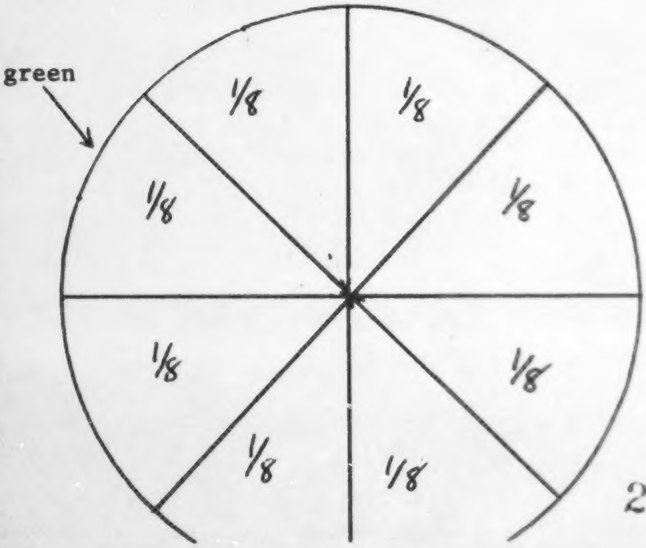
"wholes" are orange


yellow
1/2
1/2


pink
1/3
1/3
1/3


blue
1/4
1/4
1/4
1/4


brown
1/6
1/6
1/6
1/6
1/6
1/6


green
1/8
1/8
1/8
1/8
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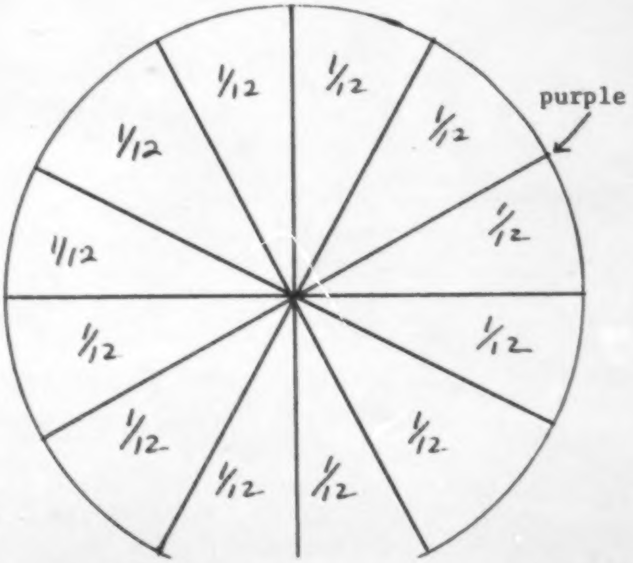
1/8


purple
1/12
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1/12
1/12
1/12

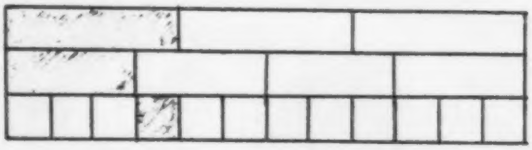
**Concept:** Multiplication of fractions, common with 1 as a numerator

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Visually "split" fraction bars or use bars equivalent to problem.

e.g.,



$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

(Use many of these types of activities and emphasize the rule -- multiply numerator by numerator and denominator by denominator.)

+ + + + + + + + + + + + + + + +

**Concept:** Multiplication of fractions, common with 1 as a numerator

**Stage of Development:** Semiconcrete/semiabstract

**Equipment:** Use transparency bars on overhead or magnetic bars on magnetic board

**Activity:** Same as above.

+ + + + + + + + + + + + + + + +

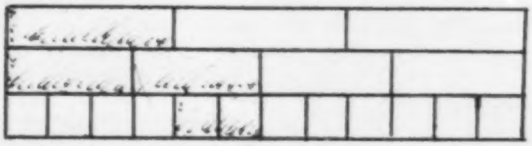
**Concept:** Multiplication of common fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Visually "split" fraction bars or use bars equivalent to problem.

e.g.,



$\frac{1}{3} \times \frac{2}{4} = \frac{2}{12}$

+ + + + + + + + + + + + + + + +

**Concept:** Multiplication of common fractions

**Stage of Development:** Semiconcrete/semiabstract

**Equipment:** Overhead transparencies and magnetic bars

**Activity:** Visually "split" fraction bars or use bars equivalent to problem as in the previous activity.

NOTE: Continue these same materials at this stage for the rest of the concepts.

+ + + + + + + + + + + + + + + +

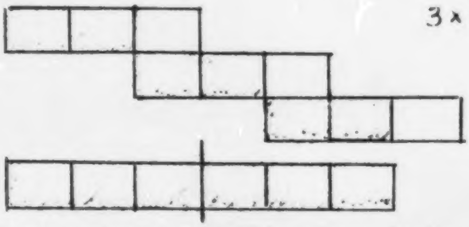
**Concept:** Multiplication of whole numbers by common fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars (two sets)

**Activity:** Line up fraction bars to prove the problem.

e.g.,



$3 \times \frac{2}{3} = \frac{6}{3} = 2$

NOTE: If students have shown proficiency in the three previous concepts and remember the rule, concrete materials should not be necessary. The following examples can be used but because of their complexity, those students having a deficiency here, should review again the preceeding material.

+ + + + + + + + + + + + + + + +

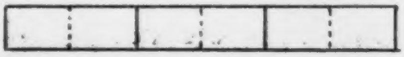
**Concept:** Multiplying improper fractions by common fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Line up fraction bars to prove the problem (visually "split").

e.g., 1 1/2 x 2/3 = 3/2 x 2/3 = 6/6 = 1



+ + + + + + + + + + + + + + + +

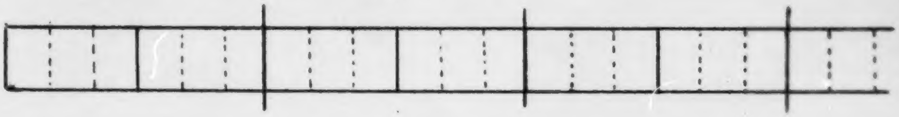
**Concept:** Multiplying improper fractions times improper fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** Line up fraction bars to prove the problem (visually "split").

(e.g., 4/3 x 5/2 = 20/6 = 3 2/6 or 3 1/3



+ + + + + + + + + + + + + + + +

**Concept:** Multiplying mixed numbers times common fractions

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** After changing the mixed numeral to an improper fraction, the procedure is the same as for improper fractions multiplied times common fractions.

+ + + + + + + + + + + + + + + +

**Concept:** Multiplying mixed numbers times mixed numbers

**Stage of Development:** Concrete

**Activity:** After changing the mixed numerals to improper fractions, the procedure is the same as for multiplying improper fractions times improper fractions.

+ + + + + + + + + + + + + + + +

# DIVISION OF FRACTIONS

## Introduction

To understand the concept of division of fractions, you need to understand the concept of multiplication of fractions. The definition of multiplication of fractions is

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

when a, b, c, and d are whole numbers, or more simply to multiply two fractions, multiply their numerators over the product of their denominators. Then simplify. Ex.

The definition of division is

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

or to divide two fractions, multiply the reciprocal of the divisor times the dividend. Ex.

$$\frac{3}{5} \div \frac{7}{8} = \frac{3}{5} \times \frac{8}{7} = \frac{24}{35}$$

Mixed numbers have to be renamed as improper fractions before they can be multiplied or divided.

Since division is defined in terms of multiplication, the same activities apply.

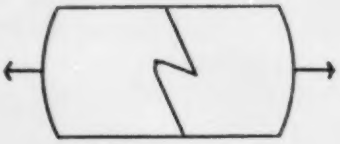
**Concept:** Division of fractions - whole number divided by common fraction

**Stage of Development:** Concrete

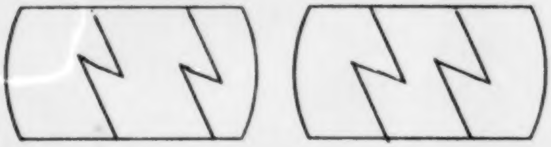
**Equipment:** Any easily divisible items such as clay, scrap material, etc.

**Activity:** Take one whole -- divide it in half. Ask, "How many pieces do you now have?"

(e.g., 1 ÷ 1/2 = 2/1 = 2

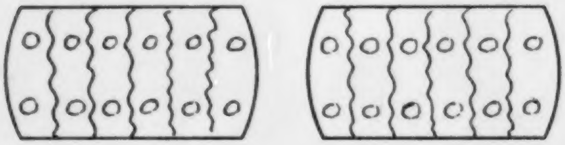


e.g., 2 ÷ 1/3 = 6/1 = 6



Have students "discover" the algorithm without too much "lecturing".

e.g., egg cartons 2 ÷ 1/6 = 12/1 = 12



+ + + + + + + + + + + + + + + +

**Concept:** Division of fractions - whole number divided by common fraction

**Stage of Development:** Semiconcrete/semiabstract

**Equipment:** Transparency materials easily divisible for overhead and magnetic materials easily divisible for magnetic board.

**Activity:** Same as above.

+ + + + + + + + + + + + + + + +

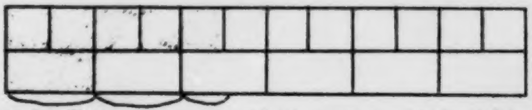
Concept: Division of common fractions

Stage of Development: Concrete

Equipment: Fraction bars

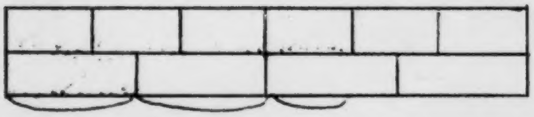
Activity: Line up fraction bars to prove the problem.

e.g., 5/12 ÷ 1/6 = 30/12 = 2 6/12 or 2 1/2



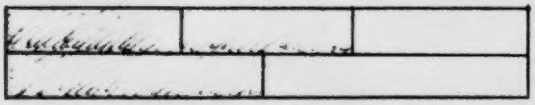
(1/6 "fits" into 5/12 twice with 1/2 of a sixth left = 2 1/2)

(e.g., 4/6 ÷ 1/4 = 16/6 = 2 4/6 or 2 2/3)



(1/4 "fits" into 4/6 twice with 2/3 of a fourth left = 2 2/3)

e.g., 2/3 ÷ 1/2 = 4/3 = 1 1/3



(1/2 "fits" into 2/3 once with 1/3 of a half left = 1 1/3)

+ + + + + + + + + + + + + + + +

Concept: Division of common fractions

Stage of Development: Semiconcrete/semiabstract

Equipment: Transparency bars or magnetic bars for overhead and magnetic boards

Activity: Same as above.

NOTE: Continue these same materials at this stage for the rest of the concepts.

+ + + + + + + + + + + + + + + +

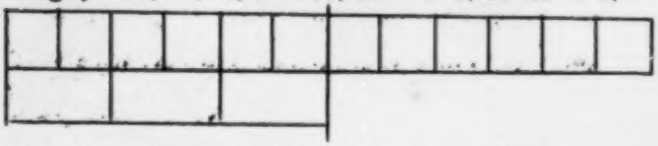
Concept: Division of improper fractions by common fractions

Stage of Development: Concrete

Equipment: Fraction bars

Activity: Line up fraction bars to prove the problem.

e.g., 11/6 ÷ 2/3 = 33/12 = 2 9/12 or 2 3/4



(2/3 "fits" into 11/6 twice with 3/4 left = 2 3/4)

+ + + + + + + + + + + + + + + +

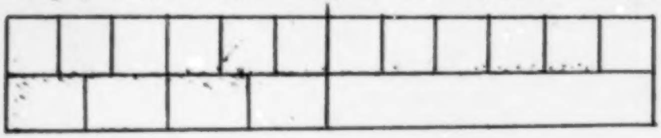
Concept: Division of improper fractions by improper fractions

Stage of Development: Concrete

Equipment: Fraction bars

Activity: Line up fraction bars to prove the problem.

e.g., 11/3 ÷ 4/2 = 22/12 = 1 10/12 or 1 5/6



(4/2 "fits" into 11/3 once with 5/6 left = 1 5/6)

NOTE: If students have shown proficiency in the above four concepts and have mastered the algorithm, concrete materials should not be necessary. The following examples can be used but because of their complexity, those students having a deficiency here should review again the preceeding material.

+ + + + + + + + + + + + + + + +

Concept: Dividing mixed numbers by common fractions

Stage of Development: Concrete

Equipment: Fraction bars

Activity: After changing the mixed numerals to improper fractions the procedure is the same as for dividing an improper fraction by a common fraction.

+ + + + + + + + + + + + + + + +

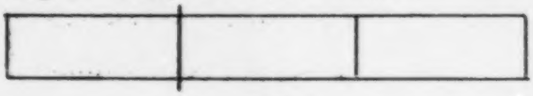
**Concept:** Common fractions divided by a whole number

**Stage of Development:** Concrete

**Equipment:** Strips of paper

**Activity:** Fold the strip of paper into the common fraction; further divide the fractional part of the paper by the whole number.

e.g., 2/3 ÷ 2 = 2/6 or 1/3



+ + + + + + + + + + + + + + + +

**Concept:** Mixed number divided by mixed number

**Stage of Development:** Concrete

**Equipment:** Fraction bars

**Activity:** After changing the mixed numerals to improper fractions the procedure is the same as for improper fractions divided by improper fractions.

+ + + + + + + + + + + + + + + + + + +

You might wonder about the emphasis on understanding what multiplying or dividing fractions mean? Isn't it easier and more important that children just be able to do the computational algorithms? This could be answered with another question, "Would your students be able to translate a real-life fraction problem into the appropriate multiplication or division number sentence without understanding what multiplication and division of fractions mean?"

## DECIMAL FRACTIONS AND DECIMALS

### Introduction

The definition of a decimal fraction is a fraction that has as its denominator a power of ten. For example

$$\frac{3}{10}, \frac{27}{100}, \frac{3}{1000}.$$

The denominator need not be written if the fraction is expressed with a decimal point and place value. The above fractions would be written

$$.3, .27, .003.$$

Another way to look at the structure of a decimal is to write the decimal in expanded form. Ex.

$$243.372 = 2 \times 100 + 4 \times 10 + 3 \times 1 + 3 \times \frac{1}{10} + 7 \times \frac{1}{100} + 2 \times \frac{1}{1000}$$

$$= 200 + 40 + 3 + \frac{3}{10} + \frac{7}{100} + \frac{2}{1000}.$$

A decimal that is written .3 represents the fraction $\frac{3}{10}$ which is defined as 3 of ten equal parts or 3 of ten equivalent sets.

To add or subtract decimals follow the same procedure used to add whole numbers. Be sure to add the same units together. The hundreds digits must be added together, the tens digits together, the ones digits together, the tenths digits together, the hundreths digits etc. Ex.

$$247.63 + 332.15 = 579.78$$

One method that can be used to insure that the proper digits are being added together is to make sure the decimal points are lined up when the numbers are written over each other vertically. Ex.

$$\begin{array}{r} 247.63 \\ \underline{+332.15} \end{array} .$$

You may want to use a lot of the same activities for decimals that you use for reinforcement of addition and subtraction of whole numbers.

To multiply two decimals together, you merely multiply the whole numbers together and then determine the placement of the decimal point in the product. The number of decimal places for the product will be the sum of the decimal places of the factors. Ex.

$$3.25 \times .002 = .00650$$

There is 2 decimal places in the first factor, 3 decimal places in the second factor therefore the product has 5 decimal places.

This can be illustrated by writing the decimals as fractions and reorganizing the computation

$$\begin{aligned} 3.25 \times .002 &= \frac{325}{100} \times \frac{2}{1000} \\ &= (325)\left(\frac{1}{100}\right) \times (2)\left(\frac{1}{1000}\right) \\ &= (325)\ (2)\left(\frac{1}{100}\right)\left(\frac{1}{1000}\right) \\ &= (325)\ (2)\left(\frac{1}{10^2}\right)\left(\frac{1}{10^3}\right) \\ &= (650)\left(\frac{1}{10^5}\right) \\ &= .00650 \end{aligned}$$

Number of decimal places

Use the activities for multiplication of whole numbers also for decimals.

To divide a whole number into a decimal divide the whole numbers being sure to plan the decimal point in the same position in the quotient as it was in the dividend. Ex.

$$\begin{array}{r|r} \multicolumn{1}{r}{} & 3.58 \\ \cline{2-2} 6 & 21.48 \\ & 18.\phantom{00} \\ \cline{2-2} & 3.4\phantom{0} \\ & 3.0\phantom{0} \\ \cline{2-2} & .48 \\ & .45 \\ \cline{2-2} \end{array}$$

The main thing to remember is the proper placement of the digits in the partial quotient when doing the long division. (Tens over tens, ones over ones, tenths over tenths etc.)

To divide by a decimal is more complex. The concept of division is to partition a set (dividend) into equivalent subsets equal in number to the divisor. The number of elements in the subset then represents the quotient. To do this the divisor has to be a whole number. Therefore when we have a division problem involving a decimal division, we must change the problem to an equivalent form where the division is a whole number. This can be accomplished by multiplying both divisor and dividend by an appropriate power of 10. Ex.

$.32\overline{)67.8}$ became $32\overline{)6780.}$ multiplying by 100.

$6.5\overline{)8.674}$ became $65\overline{)86.74}$ multiplying by 10. etc.

Don't be discouraged if your students seem to have a great deal of difficulty with this concept for it is probably the most confusing bit of computation they will have to do in school.

Good Luck.

**Concept:** Place value (decimal)

**Stage of Development:** Semiabstract

**Material:** Overhead projector

**Activity:** On the overhead projector, write 0.01 and ask students what each zero stands for and what the one stands for. Draw from them that the one represents one out of a hundred or 1/100 -- a penny is 1/100 of a dollar, and it takes 100 pennies to make one dollar. The students may wish to express their values in a different way, which is fine as long as they understand what they are saying. Write other numbers to reinforce the concept of place value. (e.g., 3.06, 1.10, 1.00 and 2.23)

+ + + + + + + + + + + + + + + +

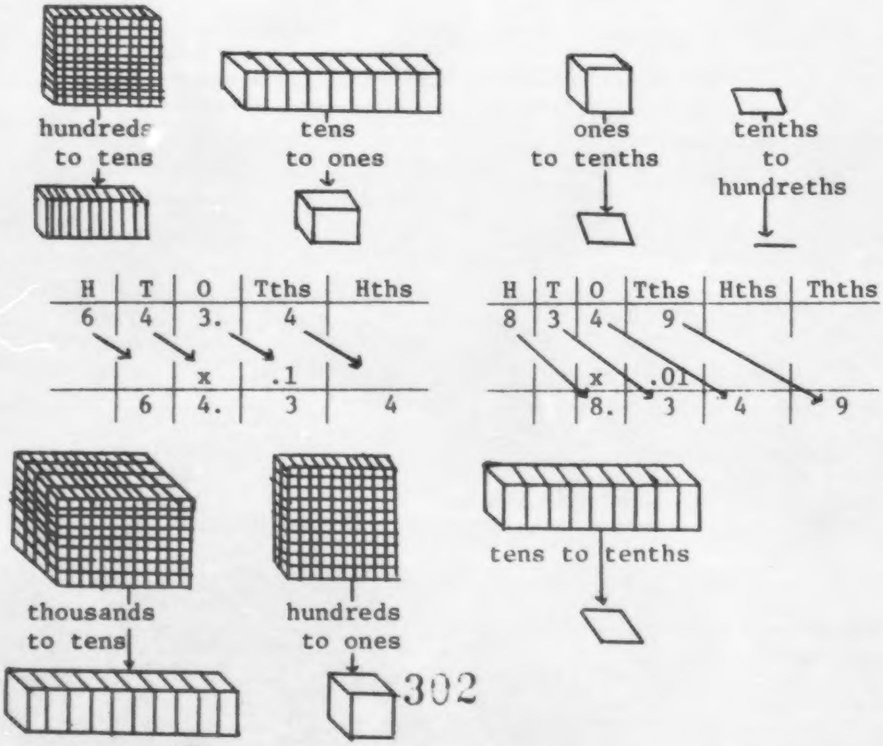
**Concept:** Multiplying by .1, .01, .001

**Stage of Development:** Semiabstract

**Material:** Overhead projector and Deinnes blocks

**Activity:** Use the Deinnes blocks to show that multiplying a quantity by one tenth is the same as taking one tenth of the quantity. The same is true for one hundredth and one thousandth.

(e.g.,


hundreds to tens
tens to ones
ones to tenths
tenths to hundreths
H | T | O | Tths | Hths
6 | 4 | 3. | 4
x | .1
6 | 4. | 3 | 4
H | T | O | Tths | Hths | Thths
8 | 3 | 4 | 9
x | .01
8. | 3 | 4 | 9
thousands to tens
hundreds to ones
tens to tenths

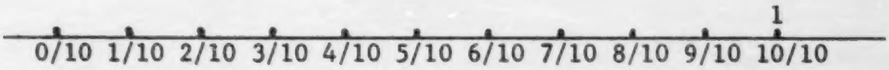
302

Concept: Decimals

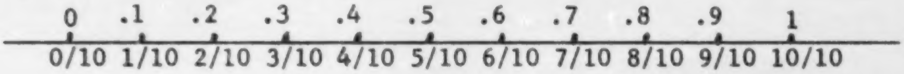
Stage of Development: Semiabstract

Materials: Chalkboard

Activity: Draw a large number line on the chalkboard. Divide the part from 0 to 1 into ten congruent segments. Have a volunteer use fractions to label the points.


1
0/10
1/10
2/10
3/10
4/10
5/10
6/10
7/10
8/10
9/10
10/10

Point out that we can also name these numbers using decimals. Instead of 1/10 we can write .1. Help students label the number line using decimals as shown below.


0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
0/10
1/10
2/10
3/10
4/10
5/10
6/10
7/10
8/10
9/10
10/10

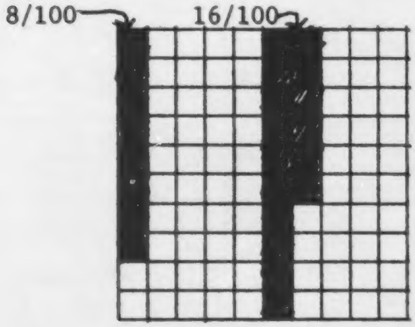
+ + + + + + + + + + + + + + + + +

Concept: Decimals naming hundredths

Stage of Development: Abstract

Materials: Graph paper

Activity: Have the students mark off a 10 x 10 square on graph paper. Then have the students shade three of the smaller square units. Ask students to describe the shaded part. Continue this questioning with examples such as the following: eight shaded units, sixteen shaded units, and etc.



8/100 = .08

16/100 = .16

Lastly, ask one of the students to tell which fraction describes the whole region. (100/100)

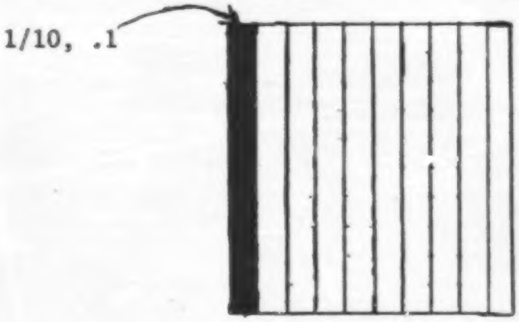
+ + + + + + + + + + + + + + + +

**Concept:** Relationship between decimals and fractions

**Stage of Development:** Semiabstract

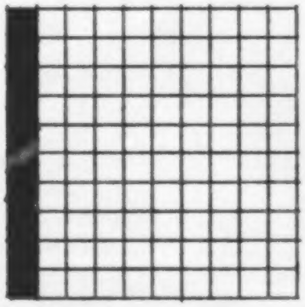
**Materials:** Overhead projector

**Activity:** Draw a 10 x 10 square on the overhead projector. Divide the square into ten congruent regions, and then shade the first region as shown:



Ask the class to state which fraction and which decimal describe the shaded region. (1/10; .1) Shade in other regions and ask students to state which fraction and decimal describe the shaded region.

Now divide the square into one hundred congruent regions. Shade in ten of the regions. Ask the class to tell which fraction and which decimal names the shaded region. (10/100; .10) Do other examples to make sure students have a good understanding of the relationship between fractions and decimals.



**NOTE:** Overlays work very nicely here, with vertical lines on one transparency and horizontal lines on the other.

+ + + + + + + + + + + + + + + +

Concept: Decimals

Stage of Development: Semiabstract

Equipment: Overhead projector, overhead projector pen, acetate roll

Activity: A set of ten objects are drawn on the acetate roll. The teacher then makes up a story which will warrant the removal of some of the elements of the set. The children then determine the fractional or decimal number associated with the remaining elements or the fractional or decimal number for those removed. Example: Three of the objects were removed (3/10 or .3), therefore, 7/10 or .7 of the objects remain.

+ + + + + + + + + + + + + + + +

Concept: Decimals

Stage of Development: Abstract

Materials: Tagboard (2" x 4")

Activity: Make several 2" x 4" cards out of tagboard. Write a fraction or a mixed numeral on each card. Each student should have a blank sheet of paper. The teacher holds up one of the cards and the students are asked to write on their paper the correct decimal form for the exposed fraction or mixed numeral.

+ + + + + + + + + + + + + + + +

Concept: Decimal

Stage of Development: Semiabstract

Equipment: Individual chalkboards

Activity: The teacher will draw on the chalkboard a number line showing points for .0, .1, .2, ... 2.0. Demonstrate addition examples such as the following on the number line.

.3 + .5 = .8; .2 + .7 = .9; .8 + .4 = 1.2

Have students to work these decimal problems as fractions, so they are able to notice the way in which addition with decimals is related to addition with fractions.

**Examples:** .3 + .5 = .8    .2 + .7 = .9    .8 + .4 = 1.2

$$\begin{array}{r} 3/10 \\ +\ 5/10 \\ \hline 8/10 \end{array} \qquad \begin{array}{r} 2/10 \\ +\ 7/10 \\ \hline 9/10 \end{array} \qquad \begin{array}{r} 8/10 \\ +\ 4/10 \\ \hline 12/10 \end{array} = 1\ 2/10$$

+ + + + + + + + + + + + + + +

**Concept:** Decimal (expanded form)

**Stage of Development:** Semiabstract

**Materials:** Individual chalkboards

**Activity:** Have each student write five decimals on his chalkboard using at least four digits in each decimal. Choose one student to read his decimals aloud. The other members of the class will write down the five decimals as he reads them. The reader then writes his five decimals on the board so the class can check what they have written. Have several other students read their decimals for the class to write. This activity could be followed with questions such as; what digit goes in the tens place? in the ones place? in the tenths place? in the hundredth place? and etc.

+ + + + + + + + + + + + + + +

**Concept:** Decimal (Place value)

**Stage of Development:** Semiabstract

**Materials:** Tagboard (2" x 4"), individual chalkboard

**Activity:** Make a deck of cards with decimal word names. The word name cards are flashed to total class and the students write the corresponding decimal on their chalkboards.

# MEASUREMENT

## General Introduction

Measurement is a form of comparison, involving the matching of an object or quantity being measured and a unit of measurement. For example, to measure a line segment, the segment is matched with an arbitrarily-chosen unit line segment. If the line segment that is being measured matches four of the chosen units, then the length of the line segment is four units (the units being inches, feet, etc.). This matching always results in a number which is known as the measure. In the above example, the measure of the line segment is four. In order to convey the full meaning of the measuring, one combines the measure with the unit, and says "four inches," "four feet," etc. This concept of measurement can be extended to include the measurement of capacity. When measuring capacity, the unit of measurement which is used in the comparison is one of the following: quarts, pints, cups, etc.

Time is something that is not measured directly. Rather than comparing a unit of time being measured, time is compared with another quantity (sand in an hourglass, numerals on a clock, shadow on a sundial) which has been calibrated by comparison with other measuring devices.

# MEASUREMENT OF CAPACITY
## (English Measurement)

## Introduction

Capacity measurement is a form of comparison. The standard units of capacity (cup, pint, quart, half gallon, gallon) are compared with the given amount of liquid. The "equals" symbol should not be used when referring to the relationship between two units of measurement. (e.g., it is not correct to say "two cups equals one pint.") The "equals" symbol is used for number sentences which give two names for the same number. For measuring, the word "make(s)" should be used instead of the "equals" symbol. (e.g., two cups make one pint.) One pint is equivalent to two cups, since they both measure the same amount of liquid, but the units of measure are not identical things, so the equality symbol is not used.

**Concept:** Capacity (cup, pint, quart, gallon)

**Stage of Development:** Concrete

**Equipment:** Cup, pint, quart, half gallon, gallon, bucket of water or sand or aquarium gravel, pitcher

**Activity:** With very young children or immature children the teacher will want to present just two units of capacity such as cup and pint. Then as the child progresses add the quart - then add the half gallon, then the gallon.

The child should be allowed to find out how many cups are in a pint by pouring full cups into the pint container until the container is full.

If you are working with the more able student you may want him to estimate how many cups are in a pint.
pints in a quart
cups in a quart
quarts in a half gallon
cups in a half gallon
pints in a half gallon
quarts in a half gallon
cups in a gallon
pints in a gallon
quarts in a gallon
half gallons in a gallon

As he estimates he keeps a record of what he thinks is the right answer. Then he actually works with the container to find out the right answer.

+ + + + + + + + + + + + + + +

**Concept:** Capacity

**Stage of Development:** Concrete

**Equipment:** Cup, pint, quart, half gallon, gallon, sand or aquarium gravel

**Activity:** Have children estimate how many cups, pints, quarts, half gallons or gallons it will take to cover a 5" square and make a pyramid. Do the same with a 6" square, etc. Change to rectangles and triangles.

+ + + + + + + + + + + + + + +

**Concept:** Capacity

**Stage of Development:** Concrete

**Equipment:** Cup, pint, quart, half gallon, gallon plus a big container

**Activity:** Have the children estimate how many cups, pints, quarts, half gallons or gallons it will take to fill the big container. Have them write down their own estimates on their own paper. Then let them actually fill up the container and count the cups used, pints used, or quarts used.

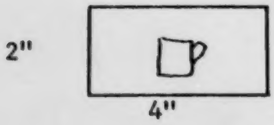
+ + + + + + + + + + + + + + +

**Concept:** Capacity (cup, pint, quart, gallon)

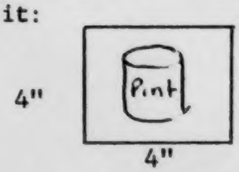
**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, flannel pictures representing a cup, pint, quart, half gallon and a gallon (Each unit of measure should be different colors of flannel.)

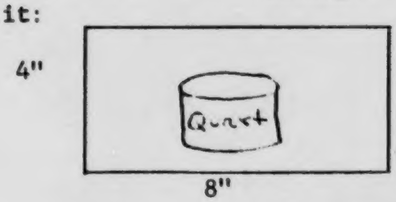
**Activity:** The flannel picture for a cup is a 2" x 4" rectangle with a drawing of a cup on it:



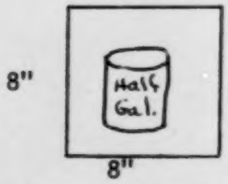
The picture for a pint is a 4" x 4" square with the following drawing on it:



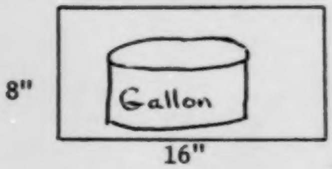
The picture for a quart is an 8" x 4" rectangle with the following drawing on it:



The picture for a half gallon is an 8" x 8" square with the following drawing on it:



The picture for a gallon is a 16" x 8" rectangle with the following drawing on it:



The picture representing the pint is placed on the flannel board. The children must estimate how many cup flannel pieces it will take to cover all of the pint piece. Someone should place the flannel pieces on the pint flannel piece until all of it is covered. One child then gives a verbal description of what has taken place. ("Two cups make a pint.") This activity is repeated as the children find the relationships between: cups and quarts, pints and quarts, cups and half gallons, pints and half gallons, quarts and half gallons, half gallons and gallons, etc.

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Stage of Development: Semiconcrete

Equipment: Magnetic board, construction paper objects representing a cup, pint, quart, half gallon and gallon. (Each unit of measure should be made from a different color of construction paper.)

Activity: The construction paper objects should be the same dimensions as the flannel pictures in the preceeding lesson. The children are to find the relationships that exist between the different measures by manipulating the construction paper objects. The relationships should be verbalized and recorded on the chalkboard.

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**Concept:** Capacity

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, overhead transparencies that represent: a pint (2" x 2" square), a quart (4" x 2" rectangle), a half gallon (4" x 4" square), a gallon (4" x 8" rectangle); flannel pieces that represent a cup (2" x 1" rectangle), a pint (2" x 2" square), a quart (4" x 2" rectangle), a half gallon (4" x 4" square)

(NOTE: Each measure should be a different color.)

**Activity:** The relationships that exist between the measures are determined by having the students place the flannel pieces on top of the transparencies. (e.g., four flannel cups will fill the pint transparency.) The relationships are written on the board as they are found.

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# MEASUREMENT OF LENGTH
## (Linear Measurement)

## Introduction

Measurement is a form of comparison. When a person asks, "How many feet are in the length of this room?" he is comparing the length of a standard unit (the foot) with the length of the room. What is actually meant is: "How many units are contained in the total set of units?" or, "How many units will fit along this length?"

The units of measure selected to compare quantities such as length are arbitrary. Standard units such as the inch, foot, and yard have been established to facilitate communication, and they have a social and historical rather than a mathematical basis. When one talks about the length of an object he is referring to how many standard units are contained in the length.

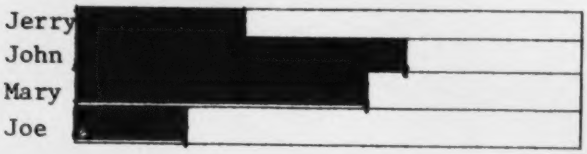
**Concept:** Linear Measurement

**Stage of Development:** Concrete

**Equipment:** Children's pencils

**Activity:** Have the children measure the length of a book, window, desk, door, etc., in pencil lengths. Make a graph for each object measured.

Book size in pencils' length



Help the children see that a standard unit of measurement is needed to measure accurately.

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**Concept:** Measurement of length

**Stage of Development:** Concrete

**Equipment:** String, paper clip necklaces, drinking straws, etc.

**Activity:** Allow children to measure various objects in the room using whatever equipment they want to use. Have them record their findings. Discuss with them what they have found. Lead them to realize that had everyone used the same unit of measurement the results would have been very similar.

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**Concept:** Measurement of length

**Stage of Development:** Concrete

**Equipment:** Foot ruler, four objects

**Activity:** Allow the children to measure the same four objects. Compare their findings. Lead the children to see the similar results.

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Concept: Linear measurement

Stage of Development: Concrete

Equipment: None

Activity: The children are told to measure the width of the classroom. They will use their feet as the unit of measure. They should count the number of feet it takes for them to walk the distance, placing each foot end to end while walking. The teacher writes down the number of feet required for each child. The distance could be referred to as 25 "Mary feet", or 21 "John feet", etc.

A graph can be made showing the different measurements. Small construction paper "footprints" are used on the graph. For the above example, Mary would have 25 "footprints", and John would have 21. The graph points out that when different units of measure are used, different answers for the same measurement are obtained. This is why it is important to use a standard unit of measurement (inch, foot, yard, etc.).

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Equipment: Foot rulers

Activity: The children measure the width of the classroom. They are to use a foot ruler as the unit of measurement. If done correctly, the measures should be the same. This is compared with the results of the activity when the children used their feet as the unit of measurement. When using their own feet as the unit of measurement, the answers depended on the size of the feet. This again emphasizes the need of a standard unit of measurement (inch, foot, yard, etc.).

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Equipment: Measuring wheel

Activity: Two children work together to measure long distances. Some things they might measure are: the length of the classroom, the length of the hallway, the length of the sidewalk in front of the school, the length of the playground, etc. An explanation of how to operate the measuring wheel is included in the section on teaching aids.

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**Equipment:** Inclined plane, toy car, ruler

**Activity:** The children are to release the car at the top of the inclined plane. This will allow the car to travel a certain distance. The students are then to measure the distance the car has traveled, using the distance between the foot of the inclined plane and the front of the car (where it has come to rest). The distance should be measured in both inches and feet. A contest can be made out of it, by seeing who can make the car travel the greatest distance.

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**Equipment:** Six foot rule (usually found with scales in school's health office)

**Activity:** The height of each child in the room is measured (correct to the nearest quarter inch). The teacher cuts out strips of paper to the following measurements: ten inches long, one inch long, one half inch long, and one fourth inch long. A chart showing each child's height can be made by laying the strips of paper end to end. This may be done vertically or horizontally. Each bar is labeled with the child's name. By comparing the heights of the bars, the following facts can be determined: the tallest child in class, the shortest child in class, and those children who are the same height. If the children are measured at the beginning of the year, and then again at the end of the year, one can see how much they have grown by comparing the heights of the bars representing their two heights.

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**Equipment:** One inch counting blocks, foot ruler, yardstick

**Activity:** The children are to determine how many one inch counting blocks are in a foot ruler. They can find out by placing the blocks end to end along the foot ruler, until the string of blocks is the same length as the ruler (the answer should be 12 blocks).

The following equivalent should be a result of the above activities:

12 inches make a foot

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Concept: Linear measurement

Stage of Development: Semiconcrete

Equipment: Magnetic board, magnetic strips (one inch and one foot long)

Activity: The magnetic strip representing one foot is placed on the magnetic board. A pupil is given a tray full of one inch magnetic strips. He then determines how many inches there are in a foot. He does this by putting the one inch strips end to end under the one foot strip, until both are the same length.

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Concept: Linear measurement

Stage of Development: Semiabstract

Equipment: Individual chalkboards, rulers

Activity: The children are told to draw line segments of different lengths on their chalkboards. They must write the measure of the line segment above the segment.

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Equipment: Overhead projector, projector pen, transparent ruler

Activity: Several line segments are drawn on the overhead projector. A student then finds the length of each segment, correct to the nearest inch. He writes the numeral that represents the number of inches in the measure on the line segment. This can be repeated with other students and other line segments.

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Stage of Development: Concrete

Equipment: Blocks, counting sticks, ruler

Activity: The children are to measure the width and the length of their desks, using different units of measurement. The teacher may ask them, "How many counting sticks wide is your desk?" "How many inches long is your desk?" The reasons for getting different answers for the same measurement should be discussed. The children then measure other objects in the classroom. It is fun to let them make up their own units of measurement.

Continue activities using various objects having specific number of units (e.g., find an object 13 inches long, etc.).

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# MEASUREMENT OF TIME

## Introduction

The teacher may want to introduce the clock face as a circular number line, for the divisions are evenly spaced around the circle. The first activity should be concerned with whole hour times. The word "o'clock" can be discussed, with the teacher pointing out that it means "of the clock". It should be explained to the children that when the larger hand is at twelve, the smaller hand signifies the specific hours and the larger hand signifies that it is exactly that hour. When teaching the half hour, the teacher should use phrases such as "half past one" or "half past two". After the children understand that there are sixty minutes in a full hour, they are ready to read times in terms of "minutes".

Concept: Time

Stage of Development: Concrete

Equipment: Paper plates, brads, tagboard hour and minute hands

Activity: The students draw a circle on their paper plates. The circle is marked off into twelve equal divisions. These divisions are labeled with the numerals, 1 - 12. Or the teacher may wish to duplicate numerals, to be cut and glued in place by children. Each plate could be premarked with a dot and child glues correct numeral over the dot. The minute and hour hands are attached to the paper plate by placing a brad through them and through the center of the plate. Each child now has a clock face. Each one is told to set his clock at a different time. The clocks are then held in the air for the teacher's observation.

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Stage of Development: Concrete

Equipment: Rubber band clock

Activity #1: The teacher demonstrates the use of the rubber band clock. The clock is set to show different times. The children read the time after each new setting.

Activity #2: A child sets the rubber band clock to show a particular time (given verbally by the teacher).

Activity #3: A time is written on the chalkboard. A child should set the rubber band clock so it shows this time.

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Stage of Development: Concrete

Equipment: Demonstration clock

Activity #1: The clock is set at a particular time and the children are asked to state this time. The teacher should begin with the time set on hour units (one o'clock, three o'clock, etc.), then go to half hours, quarter hours and finally minutes.

**Activity #2:** The teacher states a time (e.g., two o'clock). A pupil moves the hands of the clock so it shows this time.

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**Stage of Development:** Concrete

**Equipment:** 12 children in a circle, two children in the middle (one tall and one short), a minute and hour hand made of construction paper, a demonstration clock, 12 numeral cards numbered from 1 - 12

**Activity:** The 12 children join hands to form a circle. Each child has a numeral necklace around his neck (or the numerals may lay on the floor). These numerals are in consecutive order. As the teacher shows a time on her demonstration clock the two children in the middle make their construction paper hand point to the same time.

The rest of the class can take turns showing the same time the teacher shows.

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**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic apples numbered from 1 - 12, magnetic hour and minute hands

**Activity:** The teacher allows the child to show certain times on the magnetic board while the children at the seats show the same time on their paper plate clocks.

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**Concept:** Time

**Stage of Development:** Semiconcrete

**Equipment:** Flannel board, flannel clock face, flannel hour and minute hands, individual clocks

These are the directions for making the flannel clock face: Get a square piece of flannel (white is best), draw a circle on the flannel, mark the circle in twelve equal divisions, label these divisions with the numerals 1 - 12, and make an hour hand and a minute hand out of a different color of flannel. The same activities as described for the demonstration clock can then be used.

Activity #1: The flannel clock face is placed on the flannel board. The hour and minute hands are placed on the flannel clock face so that they represent a particular time (e.g., three o'clock). The students are asked to set their clocks to represent the same time.

Activity #2: A time is stated by the teacher. The students set their clocks so the verbally stated time is shown on them.

Activity #3: The children may set their clocks at any time they desire. They then hold their clocks in the air and tell the teacher what time they have.

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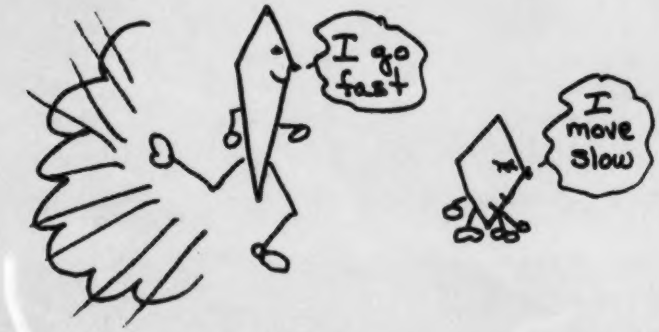
Concept: Time - movement of clock hands

Stage of Development: Semiabstract

Equipment: Chart paper, marking pen, demonstration clock

Activity: To relate concept of minute hand moving faster than hour hand, develop a class story about Mr. Speedy and Mr. Pokey. As story is written, show on clock that minute hand goes around the clock once, while hour hand goes from one numeral to another. (e.g., Mr. Speedy is tall and skinny. He is always in a hurry. He goes very fast. He can travel around the clock in 60 minutes. He is called the minute hand. Mr. Pokey is short and fat, and goes slowly. He is tired. It takes him a whole hour to move from one numeral to another. He is called the hour hand.) The story can be very simple or more involved, depending on group. The following illustrations may also help children remember the minute hand and hour hand.



**Concept:** Time

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector, overhead clock transparency

**Activity:** A circle is drawn on a transparency. It is then marked in twelve equal divisions. These divisions are labeled with the numerals, 1 - 12. An hour hand and a minute hand are constructed from cardboard. The hour and minute hands are then attached to the center of the circle with a brad. The same teaching activities as described for the demonstration clock can be used at this time.

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**Equipment:** Overhead projector, transparencies showing clock faces with different times, overhead projector pen

**Activity:** A transparency is placed on the overhead projector. The teacher states the time shown on one of the clocks. A child then marks an "x" on the clock face that shows the time stated by the teacher. This is repeated with other clock faces.

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**Stage of Development:** Semiabstract

**Equipment:** Geometric circles for each child, clock to manipulate for each child

**Activity:** Teacher folds circle in half and child folds his circle in half. Students and teacher talk about this in relationship to one half the amount of the clock face. Child needs to know that half of the circle is equivalent to 30 minutes on the clock. After folding the circle in half, then fold it in fourths and note this relationship to the quarter hours.

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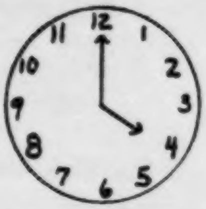
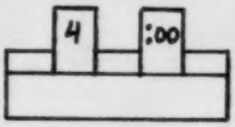
Stage of Development: Semiabstract

Equipment: Show-me-boards for each student, numeral cards for each student from 1 - 12, cards to designate times such as :15, :30, :45, :00, a demonstration clock

Activity: As the teacher shows a time on her demonstration clock the child shows the same time in his show-me card.





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Stage of Development: Semiabstract

Equipment: Chalkboard full of clocks showing different times, papers that tell different times in a paper bag

Activity: The child draws a paper and tells the time that is on the paper, then crosses out the time on the board that is the same time.

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Stage of Development: Abstract

Equipment: Sheets of mimeographed paper showing clocks with different times or just different times written without clocks

Activity: Teacher verbalizes a certain time and has the child use different color crayons to mark the clocks to coincide with the times called.

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# METRIC MEASUREMENT

## Introduction

When teaching the metric system, teach it as a system of measurement, not as a conversion of English to Metric. You will find that a firm understanding of the concept of place value is needed before your students can successfully learn the various units of measure found in the metric system. The best method of instruction for learning the metric system is to get your students actively involved in metric measurement.

A kit of materials has been prepared to assist you in your efforts to teach the metric system. No teaching activities have been listed in this book, you should request one of the metric kits when you are ready for your metric unit.

# MEASUREMENT OF MONEY

## Introduction

The teacher should not stress the learning of the names of coins unless the child has a good understanding of their value. These two concepts should go hand in hand. When naming a coin, the value of the coin should be indicated in some manner (counter frame, sets on flannel board, sets of counting objects, etc.) When this is done, a child cannot only name a coin but also has a mental image of the set that represents its value. This will make it much simpler to teach equivalent values, using different coins.

Concept: Money

Stage of Development: Concrete

Equipment: Coins, counter frame

Activity #1: Actual coins should be used. A coin is held up for all to see. A child then gives its name and shows its value on the counter frame.

Activity #2: The name of a coin is stated. One child moves counters on the counter frame to indicate the value of the coin.

Activity #3: Counters are moved on the counter frame so that they represent the value of a coin. A child gives the name of the coin. (e.g., if five counters are pushed over, then the coin is a nickel.)

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Concept: Money

Stage of Development: Semiconcrete

Equipment: Magnetic board, coins with small magnets on their backs.

Activity #1: All the magnetic coins are placed on one side of the magnetic board. A numeral that represents the value of some of these coins is written on the board. One child selects the magnetic coins that represent this value and places them under the numeral.

Activity #2: Several different magnetic coins are placed on the magnetic board. One student determines the value of these coins and writes the numeral for this value on the board, under the coins.

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**Concept:** Money

**Stage of Development:** Semiconcrete

**Equipment:** Magnetic board, magnetic discs

**Activity #1:** A grid similar to the one below can be drawn on the magnetic board:

| 50 | 25 | 10 | 5 | 1 | Value |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

**Activity #2:** Place discs on coin values. Ask the child to compute the value and write it in the right hand column.

e.g.,

| 50 | 25 | 10 | 5 | 1 | Value |
|---|---|---|---|---|---|
| ● | | | ● | ● | |
| | ● | ● | | ● | |

**Activity #3:** Ask the child to place magnetic discs on whatever coins would give them the value shown on the right.

e.g.,

| 50 | 25 | 10 | 5 | 1 | Value |
|---|---|---|---|---|---|
| | | | | | 36 |
| | | | | | 61 |

**Activity #4:** Combine the above activities on the same grid.

e.g.,

| 50 | 25 | 10 | 5 | 1 | Value |
|---|---|---|---|---|---|
| | | | | | 40 |
| ● | ● | | ● | | |

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**Concept:** Money

**Stage of Development:** Semiabstract

**Equipment:** Actual coins (penny, nickel, dime, quarter and half dollar)

**Activity #1:** Several coins of each denomination should be available. A child chooses 10¢ worth of coins (one dime, two nickels, one nickel and five pennies, or ten pennies). A different value is then stated and someone chooses coins that would represent this value.

**Activity #2:** A numeral representing the value of several coins (e.g., 78¢) is written on the board. Someone chooses coins that will represent this value. One possible answer for the example would be three quarters and three pennies.

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**Concept:** Money

**Stage of Development:** Semiabstract

**Equipment:** Actual coins (penny, nickel, dime, quarter, half dollar), cash register, various commodities

**Activity:** Price tags are placed on the commodities. Each child is given a handful of coins. The children then choose one of the commodities they wish to purchase. They must take it to the cashier, have him ring up the amount of the purchase, and give him enough coins to equal its value. The cashier must make sure that he receives the correct coins. At first, the teacher should insist that the correct coins be given to the cashier. Later the children may purposely give the cashier too much money, so he has to make change. Each student should be given an opportunity to buy a commodity.

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**Concept:** Money

**Stage of Development:** Abstract

**Equipment:** Mail order catalogs

**Activity #1:** Pupils select four items from catalogs for an "outfit" and then add up the cost.

**Activity #2:** Each pupil is given a $50.00 budget with which to buy a complete birthday outfit, from hat to shoes, underwear to coat, using the catalog.

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**Concept:** Money

**Stage of Development:** Semiabstract

**Equipment:** Overhead projector and transparencies, pointer

**Activity #1:** Show transparencies of coins (pennies, nickels, dimes, quarters, half dollars). Child uses pointer to show and name all the pennies, nickels, etc.

**Activity #2:** Looking at the transparencies the child adds the coins together for the total.

**Activity #3:** Have a transparency made of bills and coins. Child looks at the bills and coins and adds total together.

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**Concept:** Money (budgeting)

**Stage of Development:** Abstract

**Equipment:** Newspapers, paper and pencils

**Activity:** Give the children the following information on a ditto: They will earn so much money per week and they will make a budget for a month. The sheet should also designate bills for utilities, food, shelter, transportation, doctor, recreation, savings.

Have students go through want ads of newspapers to choose jobs that state how much they would earn per hour, (figure eight hour day, five day week, four weeks per month). Let each student work individually on the project. Mathematical concepts which would be used would include: addition, subtraction, multiplication, division.

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Concept: Measurement of Money-Make Change

Stage of Development: Concrete

Equipment: Commodities-Empty boxes, cans, bottles etc.
Toy cash register, money-

Activity: Mark prices on the items brought for the store. Make all the prices less than one dollar. Fill the toy cash register with coins. Give each student a toy dollar bill. Choose different students to act as the storekeeper. The storekeeper will have to make change for the items purchased. Each student should be given the opportunity to select one item from the store to purchase. They will give their dollar bill to the storekeeper and the storekeeper will have to give the student back the correct change. Both the storekeeper and the student must agree that they have given and received the correct change. Give several students the opportunity to act as storekeeper. After every student has had the opportunity to buy something from the store, allow them to return to the store with their change to make an additional purchase. Being sure to count out the change for their purchase. When finished the students could count the change they have remaining. This activity could be repeated at various times during the school year.

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## MEASUREMENT OF GEOMETRIC FIGURES

### Introduction

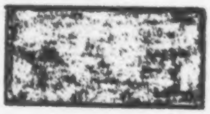
This unit is concerned with the measurement of angles in terms of degrees, perimeter of geometric figures in the unit of measure, area of geometric figures in square units of measure and volume of prisms in cubic units of measure.

A degree is defined as $\frac{1}{360}$ of a circle, therefore $90^{\circ}$ is ninety $\frac{1}{360}$ of a circle or $\frac{1}{4}$ of a circle. A protractor can be used to find the measure of an angle to the nearest degree. It will be the responsibility of the teacher to instruct the students on how to use the protractor to accurately measure angles: The measures will be recorded in terms of degrees.

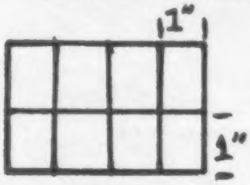
The perimeter of a geometric figure is defined as the measure of the sides of the closed figure. The perimeter of a triangle, for example, is defined as the sum of the measures of the three sides. In reference to a circle, one uses the term circumference rather than perimeter. The formula for the perimeter of a rectangle and circumference of a circle are

$$P = 2L + 2W \text{ and } C = 2\pi r \text{ or } \pi d$$

The area of a closed geometric figure is defined to be the measurable surface contained within the figure. The shaded part is the area in the figure below.



The measure of the area of a geometric figure is recorded in square units. The area of a geometric figure is then the number of square units found in its surface. These units may be centimeters, inches, feet, yards etc. A fairly accurate measure of the areas of rectangles and squares can be determined by covering their surfaces with unit squares. The number of squares would be the numerical value of the area and the units would be the same as each side of the square. Ex.



The unit squares are 1" on each side. There are 8 of them, therefore the area of the rectangle is 8 square inches.

This method of covering the surface with unit squares is not very accurate when you try to find the areas of triangles and circles. The formulas for finding the area of rectangles, triangles and circles are

$$A = L \times W,\ A = \frac{1}{2}bh, \text{ and } A = \pi r^2.$$

The area of a rectangle can be found if you know its length and width, the area of a triangle can be found by using the measure of its base and altitude and the area of a circle can be computed if you know its radius.

The volume of a geometric solid is defined as the space occupied by the solid. The volume can be computed by multiplying the area of its base times its altitude. Physically the volume of a solid could be found by placing unit cubes (1 in. x 1 in. x 1 in. or 1 cm. x 1 cm. x 1 cm. etc.) inside the solid until all the space is occupied. The number of unit cubes would be the measure of the volume. Volumes are indicated with cubic units.

*No particular activities have been written for finding perimeters, circumference, area or volumes of geometric figures. It is the responsibility of each teacher to identify geometric figures in the classroom for which students can compute area, perimeters and volumes.

Note: One method you may want to use for finding perimeters and circumferences is to lay a piece of string along the sides of the figure and then measure the length of the string.

| | |
|---|---|
| Concept: | Measurement of area |
| Stage of Development: | Concrete |
| Equipment: | Ten one inch cubes for each child |
| Activity: | Tell the children to form a field on their desk using the blocks. They are then to tell how many square units are in their field. |

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Measurement of area |
| Stage of Development: | Concrete |
| Equipment: | Yardsticks and popsicles sticks for each child |
| Activity #1: | Take the children into the school yard. Each child is to measure out a square yard on the playground. He uses the popsicle sticks for the four corners of his area. |
| Activity #2: | Allow the faster children to measure out an area of two yards by two yards. Then they are to tell the square yardage. They may work together in teams to do this. |

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Measurement of area |
| Stage of Development: | Semiconcrete |
| Equipment: | Square flannel pieces for each child |
| Activity: | The children arrange their flannel squares to form a solid surface area. They describe their surface area. The teacher or other children plot the areas on a graph. |

+ + + + + + + + + + + + + + + +

| | |
|---|---|
| Concept: | Measurement of area |
| Stage of Development: | Semiconcrete |
| Equipment: | Tagboard squares that have magnetic pieces on the back, magnetic board |
| Activity: | The teacher or the children form a solid surface area on the magnetic board. They decide how many square units are used in each solid surface area constructed. |

+ + + + + + + + + + + + + + + +

Concept: Measurement of area

Stage of Development: Semiabstract

Equipment: Overhead projector, transparency, flannel pieces

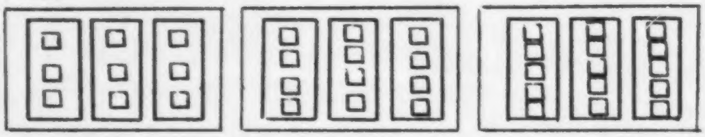
Activity: The teacher or children place as many square flannel pieces as it takes inside the surface area of the transparency. The teacher leads the children to find a quick way of determining area without always having to lay the square pieces down. (discover arrays e.g., multiplication of length times width equals area)

+ + + + + + + + + + + + + + +

Concept: Measurement of area

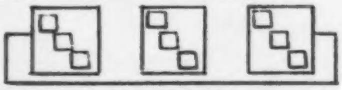
Stage of Development: Semiabstract

Equipment: Show-me-boards, cards to show different number of rows



Activity: Ask the children story problems such as --"If I had some land and I had three squares on a side, could you show me the cards that represent the story?"

The child would show



One of the children could then tell the story three squares times three squares equals nine square units.

+ + + + + + + + + + + + + + +

Concept: Measurement of area

Stage of Development: Semiabstract

Equipment: Chalkboards and chalk

Activity: Tell the children a story problem such as: "I'm thinking of a square unit of property. It has three yards on one side and four yards on the other. Show this on your chalkboard. Tell us a number story for this area."

(e.g., 3 x 4 = 12 square units) The child tells -- "If I have three units on one side and four units on the other -- in all I will have 12 square units."

+ + + + + + + + + + + + + + + +

Concept: Measurement of area

Stage of Development: Abstract

Equipment: Pencil and paper

Activity: The children have numeral and story problems on paper. They find the answers by drawing pictures or by multiplying the length times the width.

+ + + + + + + + + + + + + + + +

# INTERPRETATION OF GRAPHS

## Introduction

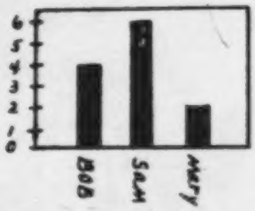
Graphs help children see relationships. It is from collecting and sorting data that comparisons are made and relationships are formed. Graphs simplify masses of figures and give details that cannot be explained easily with words. Graphs should always be accompanied by a discussion and interpretation by children. This section deals with the interpretation charts and tables, and the following graphs: pictorial, line, bar, circle and number parts.

Pictorial graphs are used to illustrate the comparison of like items in a dramatic fashion. Symbols are used within the graph to represent quantity.

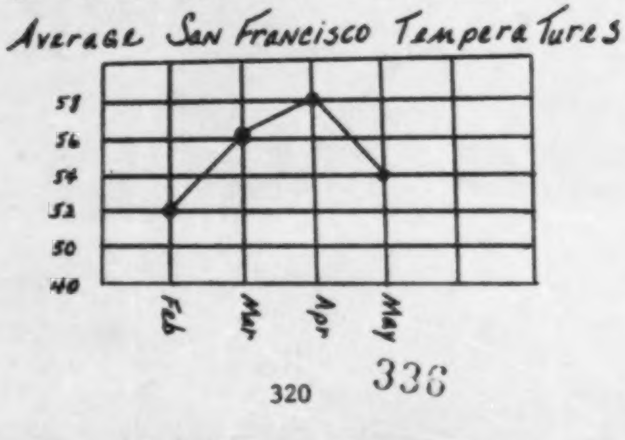
| Student | Number |
|---|---|
| Bob | ▲ ▲ ▲ ▲ |
| Sam | ▲ ▲ ▲ ▲ ▲ ▲ |
| Mary | ▲ ▲ |

The pictorial graph gives a quick visual comparison of the data.

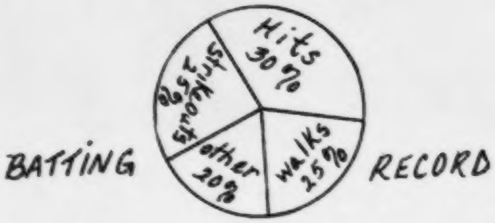
A bar graph serves much the same purpose as a pictograph. It gives a quick visual comparison of data. The height of the rectangular bar or the length of the bar represents the quantity. Same example:



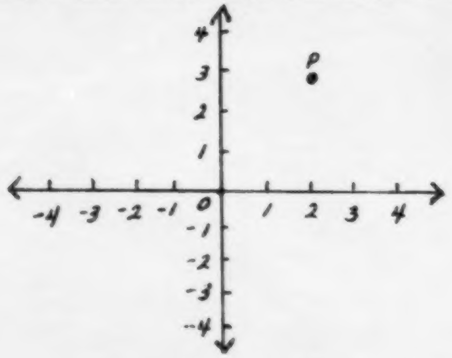
A broken line graph differs from a pictograph or a bar graph in a significant way. The categories for a pictograph or bar graph are usually discute. Example, different people, different days of the week, different schools etc. The horizontal scale for a broken line graph represents a continuous quantity such as time. When we connect two points with a line segment, we imply a continuous change from one point to the next. Ex.


Average San Francisco Temperatures

Circle graphs are useful for illustrating the relative portions of an entire item. For example, it is useful to show the relative fractions of a budget spent on different categories. You could think of the various components as slices of pie. The largest slices of pie will represent the largest quantity. A circle graph is more difficult to build than the other graphs since it requires the use of a protractor and much calculation.
Ex.



The graph used for graphing number pairs is formed by thinking of two number lines (called axis) perpendicular to each other and intersecting at their 0 points. Then a point in the plane is uniquely specified by two numbers known as its coordinates. The first coordinate specifies the distance to the right or left of the vertical axis; the second specifies the distance above or below the horizontal axis. Since point P is 2 units to the right of the vertical axis and 3 units above the horizontal axis its coordinates are (2,3).

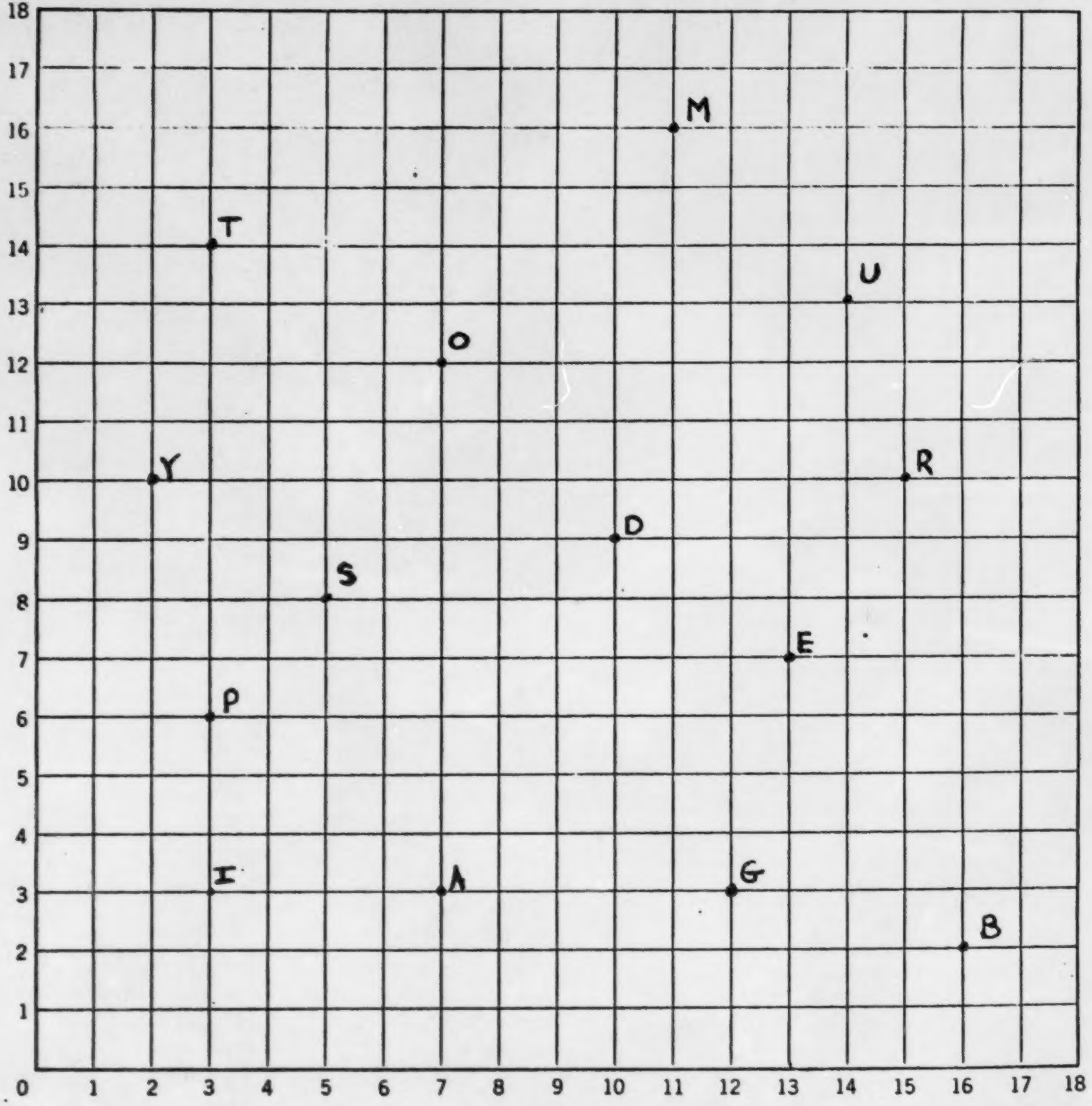


*There are sufficient quantities of meaningful activities on interpretation of graphs in the adopted text. The page numbers of specific grade levels where these activities can be found are listed below.

| GRADE | PICTORIAL | LINE | BAR | CIRCLE | NUMBER PAIRS |
|---|---|---|---|---|---|
| II | 11a,19,20 | | 11a,19,22 | | |
| III | 282--283 | | 284--286 | | |
| IV | 336--339 | | 340--343 | | 332--335 |
| V | 342--343 | 346--347<br>336--337 | 344--345 | | 340--341 |
| VI | 324--325 | 328--330 | 326--327 | 332--333 | 346--349 |

# Graphing sentences

Write the letter you find at the given coordinates below and read the secret message.



FIND THE SECRET MESSAGE:

12,3   7, 12   7, 12   10, 9

____ ____ ____ ____

2, 10   7, 12   14, 13      7, 3   15, 10   13, 7

____ ____ ____      ____ ____ ____

5, 8   11, 16 .   7, 3   15, 10   3, 14

____ ____ ____ ____ ____

## CONSTRUCTION OF GRAPHS

### Introduction Pictograph

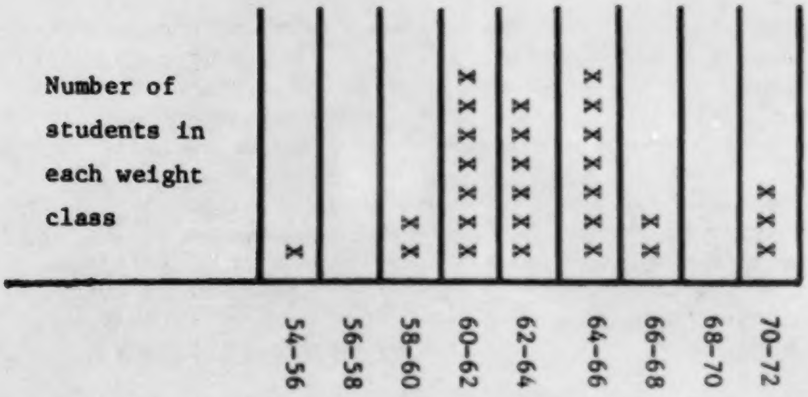
One type of pictorial graph is called a histogram. A histogram can be used to show the differences in several measurements of the same thing. To show the differences clearly your data should be arranged in certain ranges. The ranges should be equal. They should also be put on the histogram in order of size. No range in the series should be left out. It is easy to put your data in a series of equal ranges as you put it on your histogram form. The spaces on the bottom line of the histogram stand for certain ranges. You first decide on the range of measurements to put in each space. You then label each space in order of the size of the ranges.

For example, you may want to weigh each student in your class. Your data is shown below.

Weight of students

| | | | | | | |
|---|---|---|---|---|---|---|
| ~~70~~ lbs. | ~~60~~ | ~~59~~ | ~~68~~ | 56 | 58 | 65 |
| ~~68~~ | ~~64~~ | ~~61~~ | ~~$63\frac{1}{2}$~~ | 71 | 70 | 67 |
| ~~64~~ | ~~63~~ | ~~$63\frac{1}{2}$~~ | ~~66~~ | 60 | $61\frac{1}{2}$ | 65 |
| ~~63~~ | ~~65~~ | ~~$62\frac{3}{4}$~~ | ~~65~~ | 62 | 61 | 64 |

The smallest measurement is 56 and the largest measurement is 71. The difference of the extreme measurements is 15 lbs. If two lbs. is chosen às the range, you will need 8 spaces along the bottom line of your histogram.



Concept: Construction of Histogram

Stage of Development: Concrete

Equipment: Rulers; Individual Chalkboards

Activity: Give each student a ruler. Each student measures the width of your classroom correct to the nearest half inch. All the measures are written on the chalkboard. These measures will serve as the data for a histogram. Hand out the individual chalkboards. Each student is to form a histogram of the data collected. Check each students work.

+ + + + + + + + + + + + + + + +

Concept: Construction of Histogram

Stage of Development: Concrete

Equipment: Pennies; Individual Chalkboards

Activity: Give each group of students seven pennies. The students are to toss the coins in the air and record the number of heads that come up. A record is kept of each toss for 10 tosses. This data then should be used to develop a histogram. The students can use their individual chalkboards for the histogram.

+ + + + + + + + + + + + + + + +

There are many activities that will yield data that can be put in the form of histograms. Some suggestions on data that can be used for histograms are listed below.

| | |
|---|---|
| Color of childrens hair | Number in class, each color |
| Color of childrens eyes | Number in class, each color |
| Attendance | Number of children present daily |
| Days of month | Noon temperatures |
| Days of month | Maximum and minimum temperatures |
| Months of year | Number of children with birthdays each month |
| Types of pets | Number in class having each type |
| Favorite flavor of ice cream | Number of children, each flavors |
| Favorite fruit | Number of children, each kind |
| Favorite TV Program | Number of children, first choice |
| Favorite musical instrument | Number in class, each kind |
| Favorite sport | Number in class, each sport |
| Favorite singing group | Number in class, first choice |
| Favorite Pro-football Teams | Number in class, first choice |

You add to the list.

# CONSTRUCTION OF GRAPHS

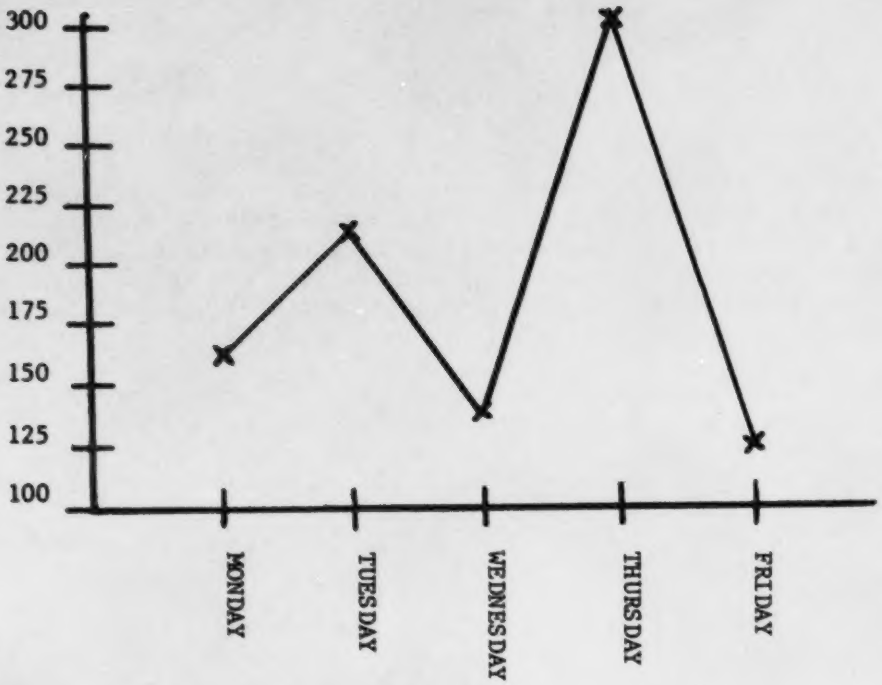
## Line Graphs

Line graphs are used to represent data that has been collected over a period of time. Examples of such data might include the temperature outside every hour for a day, the number of students buying milk in the lunchroom every day for a week or the number of cars passing through an intersection in every five minute period. A line graph of the above data makes it easy to understand.

Lets look at the procedure used to construct a line graph for the number of students buying milk in the lunchroom every day for a week.

1. The bottom line of the graph should be divided into equal parts.
2. The days of the week are put under the lines on the bottom line of the graph form.
3. The bottom line is labeled "Days of the Week".
4. The side line is divided into equal parts. Each part stands for 25 cartons of milk.
5. The lines on the side line are numbered in intervals of 25 from 100 to 300.
6. The side line is labeled "Number of Children Buying Milk".
7. Take the data for one of the days of the week.
8. Find the line on the bottom line of the graph that stands for this day of the week.
9. Draw a light line straight up from that point. The line should stop when the top of it is straight across from the number of cartons of milk sold on that particular day. Make a dot at the top of the line.
10. Put dots on the graph from each measurement you have made. After some practice, you can leave out the light line.
11. Connect the dots in order with line segments. See the completed graph listed on page 326.

LINE GRAPH



Listed below are some additional activities that can be conducted to gather data for line graphs.

1. The distance a model car travels on a level surface in certain lengths of time when it has been started at different heights on an incline.

2. The barometric pressure measured at different times in a day.

3. The number of children who can cross the street in certain times.

4. The wind speed measured at different times in a day.

5. The amount of rain fall for different months of the year.

6. The total number of major league games won for the season by the winner of the world series.

You try and think of other measures that can be used for line graphs.

Have your students draw their line graphs on their individual chalkboards.

## CONSTRUCTION OF GRAPHS

### Bar Graphs

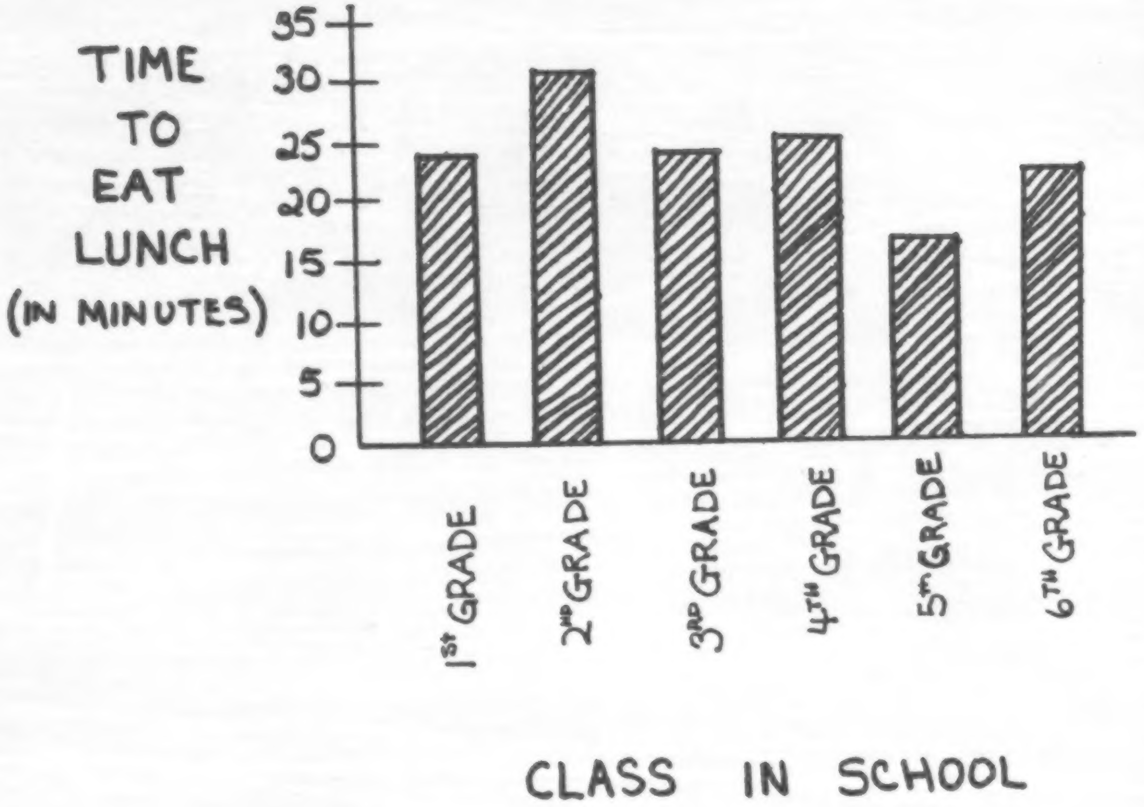
In some cases the student may want to represent the data collected by means of a bar graph. A bar graph makes the data easy to understand. An example of the kind of data that might be used for a bar graph is the time it takes for each class in school to eat their lunch in the lunchroom. The time period included the time spent in going to the lunchroom, waiting in line, picking up the food, eating, throwing out the trash and waiting to be dismissed. The data collected is shown below.

| CLASS | TIME to EAT LUNCH |
|---|---|
| 1st grade | 24 minutes |
| 2nd grade | 30 minutes |
| 3rd grade | 24 minutes |
| 4th grade | 25 minutes |
| 5th grade | 17 minutes |
| 6th grade | 22 minutes |

To put this data into the form of a bar graph you should do the following things.

1. Divide the bottom line into 6 equal parts. Each part represents a class in the school.
2. The names of the classes are put under the 6 spaces on the bottom line.
3. The bottom line is labeled "Class in School".
4. The side line is divided into equal parts. Each part stands for 5 minutes spent eating lunch.
5. The lines on the side are numbered in 5's from 0 to 35.
6. The side line is labeled "Time to Eat Lunch in Minutes".
7. Look for one class on the data chart to see how long it took that class to eat lunch.
8. Draw a bar in the space on the graph for that class. The top of the graph should be straight across from the point on the side line that shows the number of minutes it took the class to eat lunch.
9. Look at the data for the other classes. Draw bars in the other spaces. Make sure that the width of the bars is uniform as well as the distance between the bars.

The completed bar graph is shown on page 328.



Other possible sources of data are given below.

1. The number of cars that pass the school each day during a certain time period.
2. The number of students who like certain soft drinks.
3. The number of people in the class who have brown, blonde, black or red hair.
4. The number of times a colored face of a cube comes up when the cube is tossed 10 times.
5. The number of people who throw away certain foods from their lunch.
6. The winning number of home runs for that crown in the American League for the last 5 years.

You come up with different measures for which your class can draw bar graphs.

The individual chalkboards will be useful when they need something to collect data on and record their graphs.

## CONSTRUCTION of GRAPHS

### Number pairs

See the introduction on number pairs for the section on Interpretation of Graphs. A good discussion on how to construct a graph for number pairs is given there.

The only activities given in this section are those where given number pairs are graphed and connected in sequences to form a design. These activities are given in the form of a ditto master. The teacher can duplicate enough copies for each of the students in her classroom.

Note: Refer to the page numbers from the adopted text at the end of the last chapter for additional activities on constructing graphs.

## MATHEMATICAL APPLICATIONS

### Introduction

The true test of understanding any skill is its application to real life problems. This section is concerned with the application of the computational skills in the following ways:

> Computing averages, computing percentages, using formulas to solve problems, solving ratio - propostion problems, solving equations, learning the concept of rounding off numbers, working with congruent figures, solving story problems and solving problems involving money.

Page numbers where these application can be found in the adopted text are found on the next page, page 331.

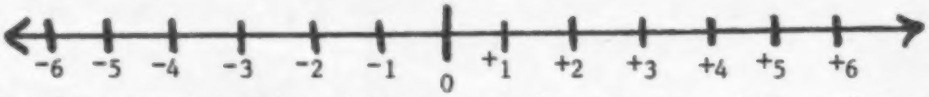
## INTEGERS

### Introduction

Integers were invented by man to enable him to subtract any two numbers in any order, to account for opposite quantities, and to solve any addition equation.

The second of these activities seems to be the easiest way to get a first understanding of the integer concept. If we designate a gain of 4 by $^{+}4$ and a loss of 4 by $^{-}4$, then we know that the net result of both is 0. Symbolized as $^{+}4 + {}^{-}4 = 0$. Reference the about data to results of consecutive plays in a football game. A gain of 4 yards followed by a loss of 4 yards has a net result of 0 yards gained.

The number line is the best method of expressing integers graphically. We merely extend it to the right for positive whole numbers and to the left for these opposites (negative whole numbers).



Positive Integers are those whole numbers found to the right of 0 on the number line. ($^{+}1, {}^{+}2, {}^{+}3, {}^{+}4, \ldots$)

Negative Integers are those whole numbers to the left of 0 on the number line. ($^{-}1, {}^{-}2, {}^{-}3, {}^{-}4, \ldots$)

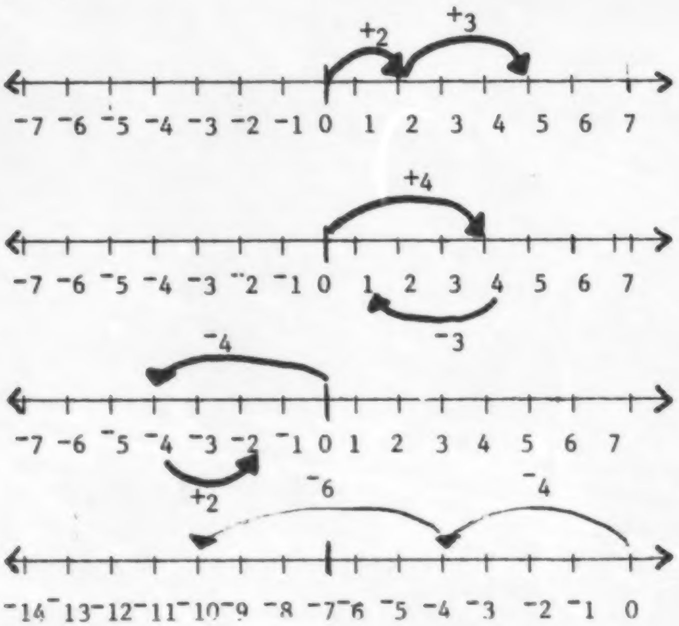
Pairs of numbers like $^{+}4$ and $^{-}4$, $^{+}5$ and $^{-}5$, $^{+}17$ and $^{-}17$ are examples of opposites. A sum of 0 is found when you add opposites together.

The best description of what happens when you add two integers is relating the addition to the net result of two consecutive plays in a football game. Thinking of positive integers as yards gained and negative integers as yards loss. There are four possible sequences of plays, a gain followed by a gain, a gain followed by a loss, a loss followed by a gain or a loss followed by a loss. Examples $^{+}2 + {}^{+}3$, $^{+}4 + {}^{-}3$, $^{-}4 + {}^{+}2$, $^{-}4 + {}^{-}6$. The net results of the following sequences is representative of their sums. $^{+}2 + {}^{+}3 = {}^{+}5$, $^{+}4 + {}^{-}3 = {}^{+}1$, $^{-}4 + {}^{+}2 = {}^{-}2$, $^{-}4 + {}^{-}6 = {}^{-}10$. A gain of two yards followed by a gain of three yards yields a net gain of 5 yards, a gain of four yards followed by a loss of three yards yields a net gain of one yard, a loss of four yards followed by a gain of two yards yields a loss of two yards, a loss of four yards followed by a loss of six yards yields a net loss of ten yards.

Another method of finding the sum of two integers is to use a number line.

To use the number line to find sums of integers begin at zero then move to the right for integers representing gains and move to the left for integers representing losses.

| Problem | Answer |
|---|---|
| ${}^{+}2 + {}^{+}3$ | ${}^{+}5$ |
| ${}^{+}4 + {}^{-}3$ | ${}^{+}1$ |
| ${}^{-}4 + {}^{+}2$ | ${}^{-}2$ |
| ${}^{-}4 + {}^{-}6$ | ${}^{-}10$ |



The concept of subtracting integers is more difficult to teach. An important concept to get across to your students is that subtracting a gain (positive integer) yeilds the same result as adding a loss (negative integer). Example-

$({}^{+}5 - {}^{+}3 = {}^{+}2) \longleftrightarrow ({}^{+}5 + {}^{-}3 = {}^{+}2)$

$({}^{-}6 - {}^{+}4 = {}^{-}10) \longleftrightarrow ({}^{-}6 + {}^{-}4 = {}^{-}10)$

$({}^{+}4 - {}^{-}2 = {}^{+}6) \longleftrightarrow ({}^{+}4 + {}^{+}2 = {}^{+}6)$

$({}^{-}4 - {}^{-}3 = {}^{-}1) \longleftrightarrow ({}^{-}4 + {}^{+}3 = {}^{-}1)$

The rule is: Subtractiong an integer is the same as adding its opposite. Therefore to subtract two integers rewrite the problems as on addition problems or use the procedure described above for addition.

Use the activities suggested in the sixth grade teachers edition on pages 334-345 for reinforcement of the concepts found in this chapter. There are some excellent card games listed under teaching suggestions on these pages.

|  | Grade 2 | Grade 3 | Grade 4 | Grade 5 | Grade 6 |
|---|---|---|---|---|---|
| Compute Averages | None | None | 214-215 | 140-141 | 104 |
| Compute Percentages | None | None | None | None | 274-287 |
| Use formulas | None | None | None | A=lw<br>308-309 | 304-305, 296<br>292, 293, 294<br>298-areas. |
|  |  |  |  |  | 310-311<br>volume |
| Ratio-Proportion | None | None | None | 208-214 | 198, 200, 201,<br>274 |
| Solve Equations | None | 156, 170<br>53-54<br>58-59<br>87 | 26-30<br>32-33 | 12-13<br>46-47 | 40-45<br>32, 56-57 |
| Rounding Numbers | None | 127-129 | 72-73, 100-101<br>180-181 | 42-43,44<br>94-95 | 8-11 |
| Congruent figures | None | None | None | 166-167 | 114, 129 |
| Story Problems | 133,135,173<br>175,182,201<br>207,211,242<br>290 | 177,207<br>257,261<br>269,300<br>145,167<br>189 | 113, 123, 178<br>209, 314, 31<br>54-55, 161<br>202, 32-33<br>155,67,75,95 | 18-19,30<br>47,53,59<br>71,84,89<br>98,122,134<br>137,172,177<br>192,207,228<br>245,254,262<br>280,290,295<br>316,348 | 22,32,38,56,68<br>81,91,97,117<br>123,139,144<br>161,172,181,188<br>196,200,207,223<br>241,249,256,282<br>288 |
| Money Problems | 153,156<br>158,183<br>208 | 126,100<br>214 | 97,98-99<br>281,243,<br>203,208,<br>187 | 56,135-139<br>67,64 | 39,97 |

Notice the grade levels where these applications are first introduced.
Plan your instruction accordingly.

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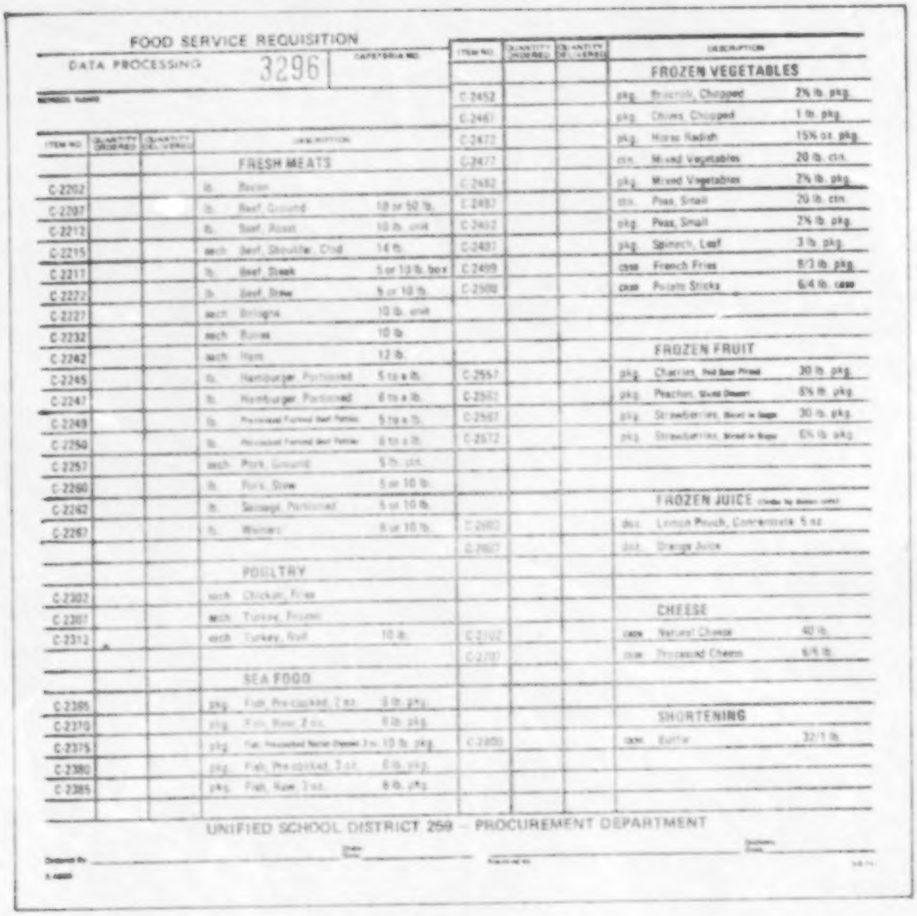


## FOOD SERVICE REQUISITION

DATA PROCESSING 3296 | CAFETERIA NO.

SCHOOL NAME

| ITEM NO. | QUANTITY ORDERED | QUANTITY DELIVERED | DESCRIPTION | | |
|---|---|---|---|---|---|
| | | | | FRESH MEATS | |
| C-2202 | | | lb. | Bacon | |
| C-2207 | | | lb. | Beef, Ground | 10 or 50 lb. |
| C-2212 | | | lb. | Beef, Roast | 10 lb. unit |
| C-2215 | | | each | Beef, Shoulder, Clod | 14 lb. |
| C-2217 | | | lb. | Beef, Steak | 5 or 10 lb. box |
| C-2222 | | | lb. | Beef, Stew | 5 or 10 lb. |
| C-2227 | | | each | Bologna | 10 lb. unit |
| C-2232 | | | each | Bones | 10 lb. |
| C-2242 | | | each | Ham | 12 lb. |
| C-2245 | | | lb. | Hamburger, Portioned | 5 to a lb. |
| C-2247 | | | lb. | Hamburger, Portioned | 6 to a lb. |
| C-2249 | | | lb. | Precooked Formed Beef Patties | 5 to a lb. |
| C-2250 | | | lb. | Precooked Formed Beef Patties | 6 to a lb. |
| C-2257 | | | each | Pork, Ground | 5 lb. ctn. |
| C-2260 | | | lb. | Pork, Stew | 5 or 10 lb. |
| C-2262 | | | lb. | Sausage, Portioned | 5 or 10 lb. |
| C-2267 | | | lb. | Weiners | 5 or 10 lb. |
| | | | | | |
| | | | | POULTRY | |
| C-2302 | | | each | Chicken, Fries | |
| C-2307 | | | each | Turkey, Frozen | |
| C-2312 | | | each | Turkey, Roll | 10 lb. |
| | | | | | |
| | | | | SEA FOOD | |
| C-2365 | | | pkg. | Fish, Precooked, 2 oz. | 6 lb. pkg. |
| C-2370 | | | pkg. | Fish, Raw, 2 oz. | 6 lb. pkg. |
| C-2375 | | | pkg. | Fish, Precooked Batter Dipped 3 oz. | 10 lb. pkg. |
| C-2380 | | | pkg. | Fish, Precooked, 3 oz. | 6 lb. pkg. |
| C-2385 | | | pkg. | Fish, Raw, 3 oz. | 6 lb. pkg. |
| | | | | | |

| ITEM NO. | QUANTITY ORDERED | QUANTITY DELIVERED | DESCRIPTION | | |
|---|---|---|---|---|---|
| | | | | FROZEN VEGETABLES | |
| C-2452 | | | pkg. | Broccoli, Chopped | 2½ lb. pkg. |
| C-2467 | | | pkg. | Chives, Chopped | 1 lb. pkg. |
| C-2472 | | | pkg. | Horse Radish | 15½ oz. pkg. |
| C-2477 | | | ctn. | Mixed Vegetables | 20 lb. ctn. |
| C-2482 | | | pkg. | Mixed Vegetables | 2½ lb. pkg. |
| C-2487 | | | ctn. | Peas, Small | 20 lb. ctn. |
| C-2492 | | | pkg. | Peas, Small | 2½ lb. pkg. |
| C-2497 | | | pkg. | Spinach, Leaf | 3 lb. pkg. |
| C-2499 | | | case | French Fries | 8/3 lb. pkg. |
| C-2500 | | | case | Potato Sticks | 6/4 lb. case |
| | | | | | |
| | | | | FROZEN FRUIT | |
| C-2557 | | | pkg. | Cherries, Red Sour Pitted | 30 lb. pkg. |
| C-2562 | | | pkg. | Peaches, Sliced Dessert | 6½ lb. pkg. |
| C-2567 | | | pkg. | Strawberries, Sliced in Sugar | 30 lb. pkg. |
| C-2572 | | | pkg. | Strawberries, Sliced in Sugar | 6½ lb. pkg. |
| | | | | | |
| | | | | FROZEN JUICE (Order by dozen units) | |
| C-2602 | | | doz. | Lemon Pouch, Concentrate 5 oz. | |
| C-2607 | | | doz. | Orange Juice | |
| | | | | | |
| | | | | CHEESE | |
| C-2702 | | | case | Natural Cheese | 40 lb. |
| C-2707 | | | case | Processed Cheese | 6/5 lb. |
| | | | | | |
| | | | | SHORTENING | |
| C-2800 | | | case | Butter | 32/1 lb. |
| | | | | | |

UNIFIED SCHOOL DISTRICT 259 — PROCUREMENT DEPARTMENT

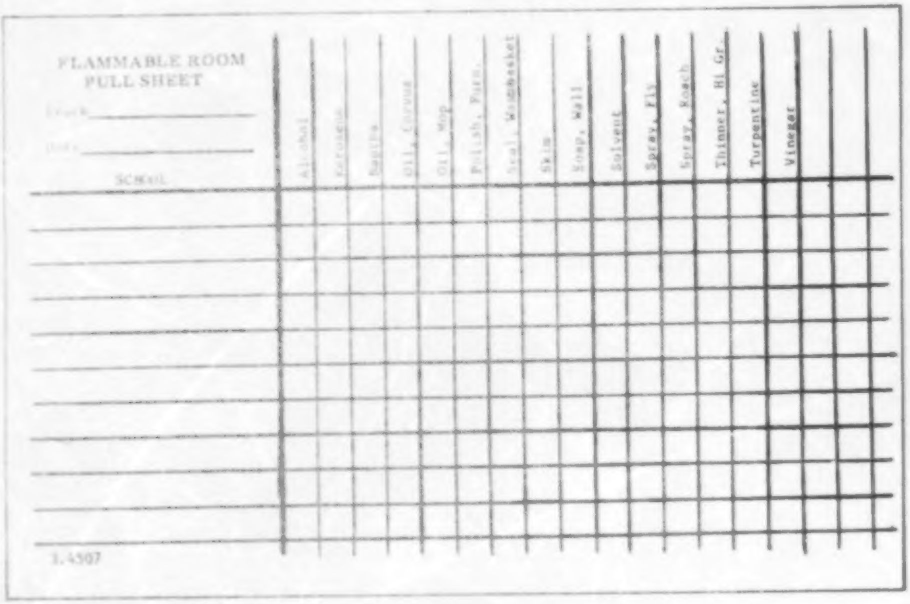
Ordered By ______ Order Date ______ Received By ______ Delivery Date ______

3.4507
BY: Division Office

USE

USE

3.4505
BY: Division Office



## FLAMMABLE ROOM PULL SHEET

Truck ______

Date ______

| SCHOOL | Alcohol | Kerosene | Naptha | Oil, Corvus | Oil, Mop | Polish, Furn. | Seal, Wastebasket | Skim | Soap, Wall | Solvent | Spray, Fly | Spray, Roach | Thinner, Hi Gr. | Turpentine | Vinegar | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | |
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3.4507

BEST COPY AVAILABLE

SCHOOL______________________________ NAME______________________________

Shade in the circle to indicate the student has demonstrated concept mastery.

GRADE K 1 2 3 4 5 6 (Circle current grade level)

**1. COMPARISONS**

- ○ Large and small
- ○ Tall and short
- ○ Heavy and light
- ○ Long
- ○ More and less

**2. POSITIONAL RELATIONSHIPS**

- ○ Under and over
- ○ Top and bottom
- ○ Front and back
- ○ High and low
- ○ Far and near
- ○ Between
- ○ Inside and outside

**3. PATTERNING**

- ○ Two shapes
- ○ Two colors
- ○ Three shapes
- ○ Three colors

**4. COUNTING BY ONES**

- ○ Rational to 10
- ○ Rational to 20
- ○ Rational to 100
- ○ Rote to 20
- ○ Rote to 100
- ○ Serial to 10
- ○ Ordinals to fifth
- ○ Ordinals to tenth
- ○ Ordinals to twentieth

**5. COUNTING BY TWOS**

- ○ Rational to 20
- ○ Rote to 50

**6. COUNTING BY FIVES**

- ○ Rational to 50
- ○ Rote to 100

**7. COUNTING BY TENS**

- ○ Rote to 100

**8. COUNTING BY HUNDREDS**

- ○ Rote to 1000

**9. RECOGNITION OF SETS**

- ○ (0-5)
- ○ (0-10)

**10. ONE-TO-ONE MATCHING**

- ○ Equivalent
- ○ Non-equivalent
- ○ One to ten
- ○ One to fifty
- ○ One to one hundred
- ○ One to one thousand

**11. RECOGNITION OF NUMERALS**

- ○ (0-10)
- ○ (0-50)
- ○ (0-100)
- ○ (0-500)
- ○ (0-1000)

**12. NUMBER WORD RECOGNITION**

- ○ One to ten
- ○ One to fifty
- ○ One to one hundred
- ○ One to five hundred
- ○ One to one thousand

**13. FORMING SETS FOR NUMERALS**

- ○ (0-10)
- ○ (0-20)

**14. MATCHING NUMERALS WITH SETS**

- ○ (0-10)
- ○ (0-20)

**15. WRITING NUMERALS**

- ○ (0-10)
- ○ (0-50)
- ○ (0-100)
- ○ (0-500)
- ○ (0-1000)

**16. ORDER OF NUMERALS**

- ○ (0-10)
- ○ (0-50)
- ○ (0-100)
- ○ (0-500)
- ○ (0-1000)
- ○ Number line

### 17. NUMBER COMPARISONS

○ Before
○ After
○ Between
○ One more
○ One less
○ Less than
○ More than
○ Even number
○ Odd number
○ Prime number
○ Number patterns

### 18. CONSERVATION OF SETS

○ (1-6)
○ (1-12)
○ (1-18)

### 19. JOINING SETS

○ (1-6)
○ (1-12)
○ (1-18)

### 20. ADDITION FACTS

○ (1-6)
○ (1-12)
○ (1-18)
○ Number line addition

### 21. SEPARATING SETS

○ (1-6)
○ (1-12)
○ (1-18)

### 22. SUBTRACTION FACTS

○ (1-6)
○ (1-12)
○ (1-18)
○ Number line subtraction

### 23. PLACE VALUE

○ Ones
○ Tens
○ Hundreds
○ Thousands
○ Millions
○ Billions
○ Expanded notations

### 24. RENAMING

○ Ones ↔ Tens
○ Tens ↔ Hundreds
○ Hundreds ↔ Thousands

### 25. ADDITION OF WHOLE NUMBERS

○ Missing addend
○ 3 addends
○ 2 digit addends (no renaming)
○ 2 digit addends (renaming)
○ 3 digit addends (no renaming)
○ 3 digit addends (renaming)
○ 4 digit addends
○ Worded problems

### 26. SUBTRACTION OF WHOLE NUMBERS

○ 2 digit (no renaming)
○ 2 digit (renaming)
○ 3 digit (no renaming)
○ 3 digit (renaming)
○ 4 digit
○ Worded problems

### 27. EQUIVALENT ~~SUBSETS~~

○ Twos
○ Threes
○ Fours
○ Fives
○ Sixes
○ Sevens
○ Eights
○ Nines

### 28. MULTIPLICATION FA

○ To 3x3
○ To 4x4
○ To 5x5
○ To 7x7
○ To 9x9
○ Number line multiplication

### 29. SYMBOLS

○ (+) ○ (-)
○ (x) ○ (÷)
○ (<) ○ (>)
○ ($) ○ (¢)
○ (%) ○ (π)
○ Triangle
○ Square
○ Rectangle
○ Circle
○ (o) ○ (=)
○ Point ○ Segme
○ Ray ○ Line
○ ⊥ lines
○ Parallel lines
○ Intersecting lines
○ Angle ○ Right a
○ Right triangle
○ Quadrilateral
○ Parallelagram
○ Rhombus

### 29. SYMBOLS, cont.

- ○ Pentagon
- ○ Hexagon
- ○ Octagon
- ○ Cube
- ○ Cone
- ○ Cylinder
- ○ Sphere
- ○ Prism

### 30. CLASSIFICATION OF NUMBERS

- ○ Composite
- ○ Prime
- ○ Common fractions
- ○ Decimals

### 31. ROMAN NUMERALS

○ (I-X) ○ (X-L)
○ (L-C) ○ (C-M)

### 32. PARTITIONING SETS

- ○ By twos
- ○ By threes
- ○ By fours
- ○ By fives
- ○ By sixes, sevens
- ○ By eights, nines

### 33. DIVISION FACTS

- ○ To 9÷3
- ○ To 16÷4
- ○ To 25÷5
- ○ To 49÷7
- ○ To 81÷9
- ○ Number line division

### 34. MULTIPLICATION OF WHOLE NUMBERS

- ○ 10 as a multiplier
- ○ 2 digit by 1 digit
- ○ 2 digit by 2 digit
- ○ 3 digit by 2 digit
- ○ 100 as a multiplier
- ○ 3 digit by 3 digit
- ○ Worded problems

### 35. DIVISION OF WHOLE NUMBERS

- ○ 1 digit divisor (no remainder)
- ○ 1 digit divisor (remainder)
- ○ 10 as a divisor
- ○ 2 digit divisor
- ○ 100 as a divisor
- ○ 3 digit divisor
- ○ Worded problems

### 36. FRACTIONS

○ (1/2) ○ (1/4)
○ (3/4) ○ (1/3)
○ (2/3)

- ○ Regions
- ○ Sets
- ○ Fractional parts of one
- ○ Renaming in simplist form
- ○ Renaming in equivalent form
- ○ Mixed numeral ↔ fractions
- ○ Order on number line

### 37. ADDITION OF FRACTIONS

- ○ Like denominators with sums less than one
- ○ Like denominators with sums greater than one
- ○ Unlike denominators
- ○ Mixed numerals

### 38. SUBTRACTION OF FRACTIONS

- ○ Common fractions with like denominators
- ○ Common fractions with unlike denominators
- ○ Whole number Common fractions
- ○ Mixed numeral Common fractions
- ○ Mixed numeral mixed numeral

### 39. MULTIPLICATION OF FRACTIONS

- ○ Common with 1 as a numerator
- ○ Common fractions
- ○ Whole number x common
- ○ Improper x common
- ○ Improper x improper
- ○ Mixed number x common
- ○ Mixed number x mixed number

### 40. DIVISION OF FRACTIONS

- ○ Whole number ÷ common
- ○ Common fractions
- ○ Improper ÷ common

### 40. DIVISION OF FRACTIONS cont.

○ Improper ÷ improper

○ Mixed number ÷ common

○ Common ÷ mixed number

○ Whole number ÷ mixed number

### 41. DECIMAL FRACTIONS

○ Tenths

○ Hundredths

○ Thousandths

○ Order on number line

### 42. DECIMALS

○ Place value

○ Expanded notation

○ Addition

○ Subtraction

○ Multiplication

○ Division

### 43. ENGLISH MEASUREMENT

○ Cup ○ Pint

○ Quart ○ Gallon

○ Inch ○ Foot

○ Yard

○ Ounce ○ Pound

### 44. MEASUREMENT OF TIME

○ Hour

○ Half-hour

○ Quarter time

○ Exact time

### 45. METRIC MEASUREMENT

○ Metre ○ Litre

○ Gram

○ Centi- cm, cg, cl

○ Deci- dm, dg, dl

○ Deka- dam, dag, dal

○ Kilo- km, kg, kl

○ Milli- mm, mg, ml

○ Hecto- hm, hg, hl

### 46. MEASUREMENT OF MONEY

○ Penny ○ Nickel

○ Dime ○ Quarter

○ Half dollar

○ Dollar

○ Knows values

○ Makes change

### 47. MEASUREMENT OF GEOMETRIC FIGURES

○ Angles in degrees

○ Perimeter of polygons

○ Circumference of circles

○ Area of rectangles

○ Area of triangles

○ Area of circles

○ Volume of prism

### 48. INTERPRETATION OF GRAPHS

○ Charts and tables

○ Pictorial

○ Line

○ Bar

○ Circle

○ Number pairs

### 49. CONSTRUCTION OF GRAPHS

○ Charts and tables

○ Pictorial

○ Line

○ Bar

○ Circle

○ Number pairs

### 50. MATHEMATICAL APPLICATIONS

○ Compute averages

○ Compute percentages

○ Use formulas

○ Ratio--Proportion problems

○ Solve equations

○ Rounding numbers

○ Congruent figures

○ Story problems

○ Money story problems

### 51. INTEGERS

○ Positive

○ Negative

○ Opposites

○ Addition

○ Subtraction